Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.4-Improper/1.2.4.2-d-x-^m-a-x^q+b-x^n+c-x^-2-n-q-^p

Nasser M. Abbasi

May 14, 2020 Compiled on May 14, 2020 at 12:59am

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [140]. This is test number [50].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (140)	% 0. (0)
Mathematica	% 99.29 (139)	% 0.71 (1)
Maple	% 97.14 (136)	% 2.86 (4)
Maxima	% 15.71 (22)	% 84.29 (118)
Fricas	% 92.14 (129)	% 7.86 (11)
Sympy	% 47.86 (67)	% 52.14 (73)
Giac	% 62.14 (87)	% 37.86 (53)

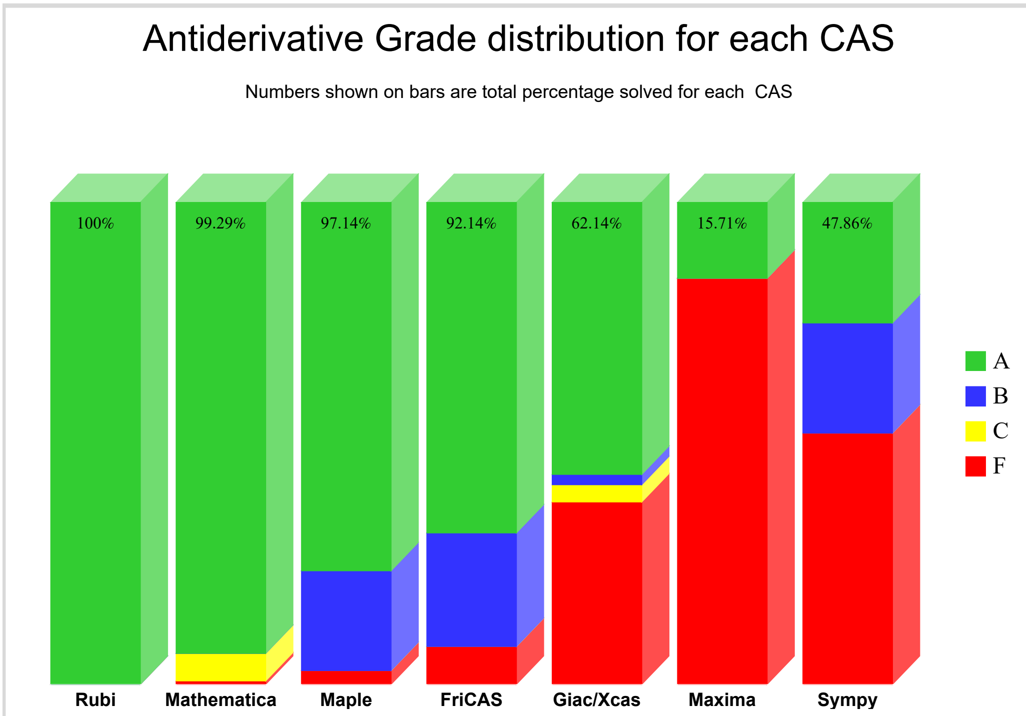
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

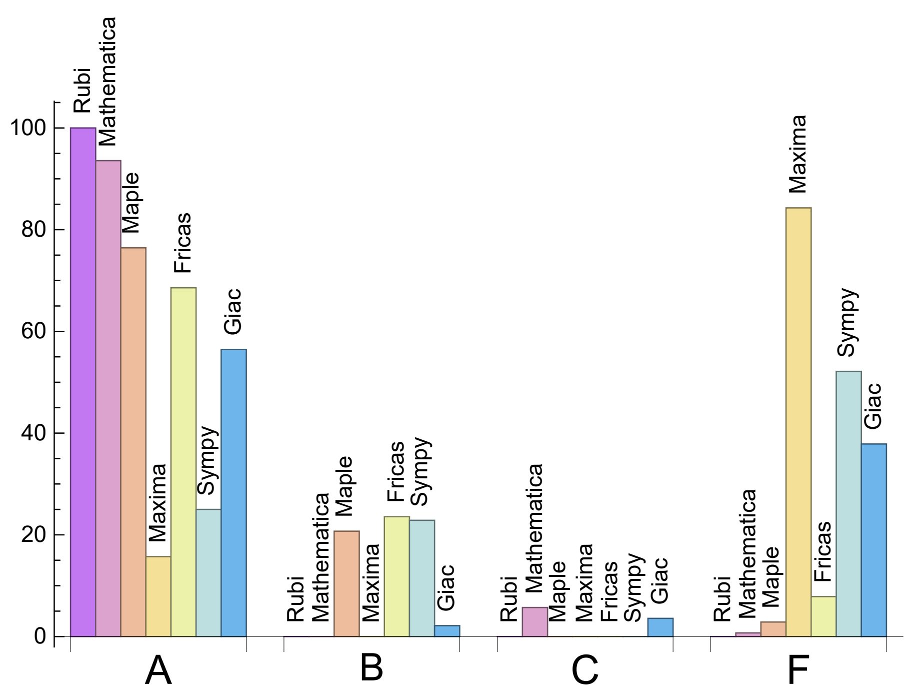
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	93.57	0.	5.71	0.71
Maple	76.43	20.71	0.	2.86
Maxima	15.71	0.	0.	84.29
Fricas	68.57	23.57	0.	7.86
Sympy	25.	22.86	0.	52.14
Giac	56.43	2.14	3.57	37.86

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.2	139.59	1.	103.5	1.
Mathematica	0.23	135.97	1.03	109.	0.99
Maple	0.01	236.36	1.43	143.	1.33
Maxima	1.15	40.73	1.08	26.5	1.09
Fricas	1.73	1036.53	6.84	626.	6.38
Sympy	7.06	613.04	4.55	267.	3.54
Giac	3.46	390.94	2.8	103.	1.35

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{104, 122\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

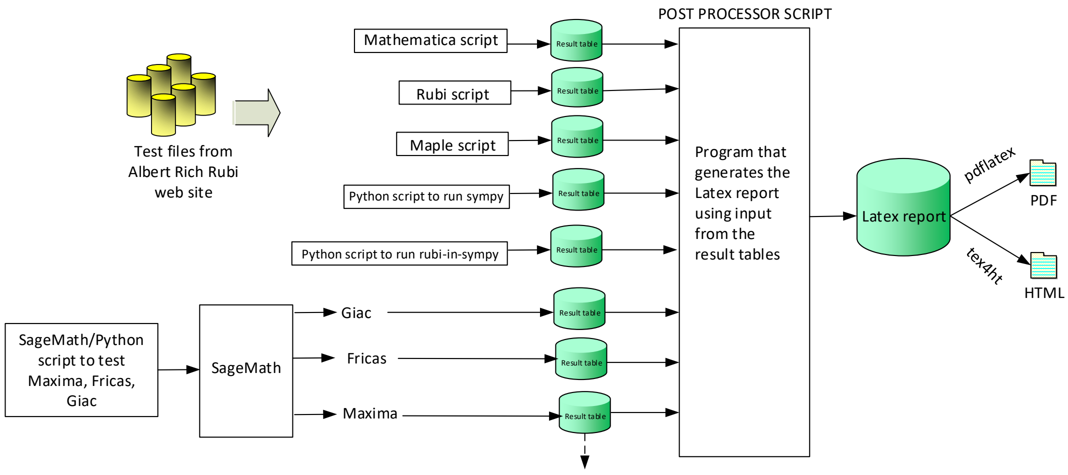
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 108, 109, 111, 113, 115, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { }

C grade: { 105, 107, 110, 112, 114, 116, 117, 119 }

F grade: { 140}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 83, 84, 85, 86, 87, 88, 91, 93, 95, 96, 97, 106, 107, 108, 109, 111, 113, 114, 115, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 19, 24, 25, 26, 27, 28, 35, 37, 47, 48, 65, 72, 79, 81, 89, 90, 92, 94, 98, 99, 100, 101, 102, 103, 105, 110, 112, 118, 119 }

C grade: { }

F grade: { 104, 121, 122, 140 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 137

B grade: { }

C grade: { }

F grade: { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 72, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 106, 108, 109, 111, 113, 115, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 60, 72, 79, 81, 83, 85, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 118 }

C grade: { }

F grade: { 104, 105, 107, 110, 112, 114, 116, 117, 119, 122, 140 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 81, 83, 85, 87, 90, 92, 94, 96, 98, 100, 102 }

B grade: { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97, 99, 101, 103 }

C grade: { }

F grade: { 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 50, 51, 52, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97, 99, 101, 103, 106, 113, 126, 127, 128, 129, 130, 131, 132, 139 }

B grade: { 65, 72, 111 }

C grade: { 79, 81, 83, 85, 87 }

F grade: { 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 60, 61, 62, 63, 64, 90, 92, 94, 96, 98, 100, 102, 104, 105, 107, 108, 109, 110, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 140 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.007	0.002	0.	1.099	1.279	0.058	1.099

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.007	0.002	0.	1.158	1.301	0.058	1.101

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.004	0.	0.	1.076	1.365	0.058	1.081

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.006	0.001	0.001	1.071	1.442	0.057	1.083

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	39	15	22
normalized size	1	1.	1.	0.85	1.1	1.95	0.75	1.1
time (sec)	N/A	0.006	0.001	0.002	1.118	1.468	0.056	1.075

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	116	48	62
normalized size	1	1.	1.	0.83	1.09	2.15	0.89	1.15
time (sec)	N/A	0.054	0.01	0.001	1.165	1.268	0.071	1.071

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	115	49	62
normalized size	1	1.	1.	0.83	1.09	2.13	0.91	1.15
time (sec)	N/A	0.029	0.007	0.	1.127	1.289	0.07	1.081

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	65	112	48	62
normalized size	1	1.	1.	0.83	1.2	2.07	0.89	1.15
time (sec)	N/A	0.027	0.006	0.	1.337	1.365	0.073	1.096

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	107	49	62
normalized size	1	1.	1.	0.83	1.09	1.98	0.91	1.15
time (sec)	N/A	0.032	0.006	0.002	1.125	1.425	0.07	1.099

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	107	48	62
normalized size	1	1.	1.	0.83	1.09	1.98	0.89	1.15
time (sec)	N/A	0.032	0.007	0.	1.153	1.455	0.072	1.072

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	132	0	653	381	116
normalized size	1	1.	0.94	1.48	0.	7.34	4.28	1.3
time (sec)	N/A	0.094	0.11	0.003	0.	1.59	0.784	1.083

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	73	101	0	537	306	90
normalized size	1	1.	1.04	1.44	0.	7.67	4.37	1.29
time (sec)	N/A	0.059	0.064	0.003	0.	1.688	0.647	1.083

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	427	216	74
normalized size	1	1.	1.02	1.	0.	7.62	3.86	1.32
time (sec)	N/A	0.04	0.032	0.001	0.	1.559	0.302	1.105

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	277	124	46
normalized size	1	1.	1.12	1.03	0.	8.15	3.65	1.35
time (sec)	N/A	0.026	0.006	0.002	0.	1.571	0.201	1.113

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	494	564	84
normalized size	1	1.	0.98	1.	0.	7.97	9.1	1.35
time (sec)	N/A	0.047	0.073	0.006	0.	1.762	1.927	1.083

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	112	0	626	862	107
normalized size	1	1.	0.95	1.38	0.	7.73	10.64	1.32
time (sec)	N/A	0.098	0.079	0.006	0.	1.776	3.507	1.093

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	102	150	0	798	1525	142
normalized size	1	1.	0.98	1.44	0.	7.67	14.66	1.37
time (sec)	N/A	0.146	0.138	0.007	0.	1.946	5.173	1.132

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	131	214	0	979	2105	184
normalized size	1	1.	0.96	1.56	0.	7.15	15.36	1.34
time (sec)	N/A	0.203	0.114	0.009	0.	2.039	8.804	1.093

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	132	352	0	1760	842	217
normalized size	1	1.	0.88	2.35	0.	11.73	5.61	1.45
time (sec)	N/A	0.159	0.193	0.008	0.	1.598	1.625	1.116

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	109	209	0	1374	729	169
normalized size	1	1.	0.96	1.83	0.	12.05	6.39	1.48
time (sec)	N/A	0.103	0.153	0.009	0.	1.658	1.209	1.097

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	81	97	0	826	280	119
normalized size	1	1.	1.21	1.45	0.	12.33	4.18	1.78
time (sec)	N/A	0.039	0.094	0.006	0.	1.597	0.756	1.117

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	740	252	103
normalized size	1	1.	1.05	1.06	0.	11.21	3.82	1.56
time (sec)	N/A	0.038	0.068	0.002	0.	1.53	0.705	1.14

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	745	265	103
normalized size	1	1.	1.06	1.03	0.	11.29	4.02	1.56
time (sec)	N/A	0.035	0.079	0.003	0.	1.551	0.722	1.106

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	107	237	0	1685	2236	170
normalized size	1	1.	0.99	2.19	0.	15.6	20.7	1.57
time (sec)	N/A	0.148	0.236	0.012	0.	2.092	7.645	1.131

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	131	328	0	2049	2672	231
normalized size	1	1.	0.89	2.22	0.	13.84	18.05	1.56
time (sec)	N/A	0.198	0.305	0.013	0.	2.612	11.941	1.089

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	175	418	0	2606	4083	309
normalized size	1	1.	0.87	2.07	0.	12.9	20.21	1.53
time (sec)	N/A	0.25	0.428	0.016	0.	3.117	19.174	1.121

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	218	515	0	2989	4774	381
normalized size	1	1.	0.87	2.04	0.	11.86	18.94	1.51
time (sec)	N/A	0.323	0.341	0.016	0.	4.108	31.024	1.107

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	272	619	0	3563	6181	468
normalized size	1	1.	0.86	1.95	0.	11.2	19.44	1.47
time (sec)	N/A	0.392	0.413	0.018	0.	5.725	47.472	1.123

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	180	310	0	898	0	382
normalized size	1	1.	0.7	1.21	0.	3.49	0.	1.49
time (sec)	N/A	0.588	0.241	0.007	0.	1.736	0.	1.182

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	150	265	0	737	0	311
normalized size	1	1.	0.73	1.29	0.	3.6	0.	1.52
time (sec)	N/A	0.37	0.193	0.008	0.	1.646	0.	1.211

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	119	167	0	591	0	224
normalized size	1	1.	0.73	1.02	0.	3.63	0.	1.37
time (sec)	N/A	0.058	0.22	0.006	0.	1.615	0.	1.144

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	100	146	0	502	0	169
normalized size	1	1.	0.84	1.23	0.	4.22	0.	1.42
time (sec)	N/A	0.078	0.146	0.004	0.	1.669	0.	1.166

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	134	126	0	1457	0	0
normalized size	1	1.	0.77	0.73	0.	8.42	0.	0.
time (sec)	N/A	0.126	0.11	0.004	0.	1.842	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	131	174	0	1489	0	0
normalized size	1	1.	0.76	1.01	0.	8.61	0.	0.
time (sec)	N/A	0.124	0.127	0.006	0.	1.963	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	112	207	0	517	0	0
normalized size	1	1.	0.98	1.82	0.	4.54	0.	0.
time (sec)	N/A	0.148	0.1	0.006	0.	1.692	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	131	234	0	609	0	0
normalized size	1	1.	0.85	1.51	0.	3.93	0.	0.
time (sec)	N/A	0.256	0.144	0.006	0.	1.841	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	160	387	0	757	0	0
normalized size	1	1.	0.78	1.89	0.	3.69	0.	0.
time (sec)	N/A	0.386	0.225	0.006	0.	2.129	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	236	649	0	1600	0	703
normalized size	1	1.	0.56	1.54	0.	3.79	0.	1.67
time (sec)	N/A	1.203	0.426	0.008	0.	2.031	0.	1.294

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	197	479	0	1299	0	579
normalized size	1	1.	0.54	1.32	0.	3.57	0.	1.59
time (sec)	N/A	1.039	0.267	0.009	0.	1.985	0.	1.302

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	180	431	0	1099	0	493
normalized size	1	1.	0.62	1.5	0.	3.82	0.	1.71
time (sec)	N/A	0.519	0.242	0.009	0.	1.835	0.	1.325

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	163	289	0	878	0	383
normalized size	1	1.	0.82	1.46	0.	4.43	0.	1.93
time (sec)	N/A	0.178	0.182	0.006	0.	1.695	0.	1.273

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	132	265	0	725	0	313
normalized size	1	1.	0.8	1.61	0.	4.39	0.	1.9
time (sec)	N/A	0.128	0.064	0.006	0.	1.646	0.	1.28

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	166	222	0	1825	0	0
normalized size	1	1.	0.73	0.98	0.	8.04	0.	0.
time (sec)	N/A	0.255	0.238	0.005	0.	2.562	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	158	254	0	1735	0	0
normalized size	1	1.	0.72	1.16	0.	7.92	0.	0.
time (sec)	N/A	0.242	0.184	0.006	0.	2.218	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	162	338	0	1737	0	0
normalized size	1	1.	0.74	1.54	0.	7.93	0.	0.
time (sec)	N/A	0.238	0.199	0.006	0.	2.242	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	175	435	0	1871	0	0
normalized size	1	1.	0.68	1.69	0.	7.28	0.	0.
time (sec)	N/A	0.351	0.307	0.007	0.	2.5	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	141	501	0	743	0	0
normalized size	1	1.	0.72	2.54	0.	3.77	0.	0.
time (sec)	N/A	0.363	0.117	0.009	0.	2.341	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	177	534	0	898	0	0
normalized size	1	1.	0.71	2.14	0.	3.61	0.	0.
time (sec)	N/A	0.504	0.176	0.008	0.	2.71	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	105	144	0	514	0	0
normalized size	1	1.	0.73	1.01	0.	3.59	0.	0.
time (sec)	N/A	0.174	0.137	0.006	0.	1.569	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	89	88	0	429	0	146
normalized size	1	1.	0.86	0.85	0.	4.17	0.	1.42
time (sec)	N/A	0.078	0.054	0.006	0.	1.591	0.	1.156

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	65	0	298	0	50
normalized size	1	1.	0.93	0.92	0.	4.2	0.	0.7
time (sec)	N/A	0.037	0.041	0.006	0.	1.546	0.	1.159

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	70	66	0	300	0	80
normalized size	1	1.	1.56	1.47	0.	6.67	0.	1.78
time (sec)	N/A	0.016	0.021	0.005	0.	1.616	0.	1.157

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	89	88	0	444	0	0
normalized size	1	1.	1.16	1.14	0.	5.77	0.	0.
time (sec)	N/A	0.054	0.056	0.006	0.	1.732	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	112	152	0	527	0	0
normalized size	1	1.	0.94	1.28	0.	4.43	0.	0.
time (sec)	N/A	0.149	0.096	0.006	0.	1.791	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	183	283	0	1316	0	0
normalized size	1	1.	0.7	1.08	0.	5.02	0.	0.
time (sec)	N/A	0.506	0.258	0.009	0.	2.276	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	141	199	0	1026	0	263
normalized size	1	1.	0.7	0.99	0.	5.1	0.	1.31
time (sec)	N/A	0.305	0.159	0.006	0.	2.09	0.	1.17

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	112	166	0	873	0	149
normalized size	1	1.	0.73	1.08	0.	5.71	0.	0.97
time (sec)	N/A	0.175	0.123	0.006	0.	2.159	0.	1.192

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	53	0	150	0	61
normalized size	1	1.	0.92	1.32	0.	3.75	0.	1.52
time (sec)	N/A	0.04	0.074	0.004	0.	1.985	0.	1.192

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	0	151	0	61
normalized size	1	1.	0.92	1.33	0.	3.87	0.	1.56
time (sec)	N/A	0.04	0.024	0.003	0.	1.946	0.	1.165

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	109	164	0	869	0	0
normalized size	1	1.	1.16	1.74	0.	9.24	0.	0.
time (sec)	N/A	0.068	0.123	0.006	0.	2.065	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	138	201	0	1041	0	0
normalized size	1	1.	0.96	1.4	0.	7.23	0.	0.
time (sec)	N/A	0.163	0.106	0.006	0.	1.876	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	181	292	0	1328	0	0
normalized size	1	1.	0.87	1.4	0.	6.35	0.	0.
time (sec)	N/A	0.287	0.167	0.007	0.	2.208	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	225	340	0	1550	0	0
normalized size	1	1.	0.83	1.25	0.	5.72	0.	0.
time (sec)	N/A	0.452	0.265	0.009	0.	2.913	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	272	446	0	1901	0	0
normalized size	1	1.	0.79	1.3	0.	5.54	0.	0.
time (sec)	N/A	0.621	0.287	0.008	0.	4.085	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	77	0	161	280	144
normalized size	1	1.	0.92	2.08	0.	4.35	7.57	3.89
time (sec)	N/A	0.013	0.029	0.003	0.	1.313	1.09	1.112

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.008	0.002	0.002	1.127	1.051	0.056	1.098

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.007	0.002	0.	1.072	1.087	0.056	1.091

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.004	0.	0.	1.114	1.049	0.056	1.1

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	39	15	22
normalized size	1	1.	1.	0.85	1.1	1.95	0.75	1.1
time (sec)	N/A	0.006	0.001	0.	1.131	1.23	0.056	1.1

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	46	17	27
normalized size	1	1.	1.	0.86	1.1	2.19	0.81	1.29
time (sec)	N/A	0.008	0.002	0.003	1.142	1.224	0.089	1.121

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	42	12	22
normalized size	1	1.	1.	0.94	1.22	2.33	0.67	1.22
time (sec)	N/A	0.007	0.002	0.003	1.112	1.191	0.248	1.091

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	300	0	602	1377	539
normalized size	1	1.	0.91	3.95	0.	7.92	18.12	7.09
time (sec)	N/A	0.046	0.079	0.006	0.	1.41	3.99	1.127

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	117	51	62
normalized size	1	1.	1.	0.83	1.09	2.17	0.94	1.15
time (sec)	N/A	0.036	0.007	0.001	1.119	1.015	0.07	1.086

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	59	116	46	62
normalized size	1	1.	0.89	0.83	1.09	2.15	0.85	1.15
time (sec)	N/A	0.054	0.009	0.001	1.09	1.123	0.07	1.09

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	65	115	51	62
normalized size	1	1.	1.	0.83	1.2	2.13	0.94	1.15
time (sec)	N/A	0.026	0.008	0.002	1.12	1.118	0.07	1.098

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	59	109	46	62
normalized size	1	1.	0.89	0.83	1.09	2.02	0.85	1.15
time (sec)	N/A	0.046	0.009	0.002	1.022	1.309	0.069	1.089

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	55	99	48	58
normalized size	1	1.	1.	0.86	1.12	2.02	0.98	1.18
time (sec)	N/A	0.027	0.005	0.001	1.128	1.099	0.072	1.078

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	93	142	0	675	391	124
normalized size	1	1.	0.93	1.42	0.	6.75	3.91	1.24
time (sec)	N/A	0.122	0.094	0.005	0.	1.329	1.892	1.1

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	250	467	0	3217	194	5466
normalized size	1	1.	1.23	2.3	0.	15.85	0.96	26.93
time (sec)	N/A	0.602	0.172	0.023	0.	1.54	2.207	2.487

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	556	316	101
normalized size	1	1.	0.96	1.37	0.	6.86	3.9	1.25
time (sec)	N/A	0.087	0.048	0.003	0.	1.278	1.541	1.095

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	202	343	0	2168	129	4593
normalized size	1	1.	1.13	1.92	0.	12.11	0.72	25.66
time (sec)	N/A	0.227	0.145	0.013	0.	1.412	1.604	2.336

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	443	223	80
normalized size	1	1.	0.98	0.95	0.	7.03	3.54	1.27
time (sec)	N/A	0.067	0.028	0.003	0.	1.316	0.813	1.089

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	165	208	0	1206	75	5350
normalized size	1	1.	1.1	1.39	0.	8.04	0.5	35.67
time (sec)	N/A	0.094	0.098	0.011	0.	1.522	0.768	2.206

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	290	131	47
normalized size	1	1.	1.08	1.	0.	8.06	3.64	1.31
time (sec)	N/A	0.043	0.009	0.	0.	1.539	0.445	1.116

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	129	116	0	1323	87	1365
normalized size	1	1.	0.86	0.77	0.	8.82	0.58	9.1
time (sec)	N/A	0.08	0.083	0.01	0.	1.571	0.896	1.359

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	113	66	0	510	253	92
normalized size	1	1.	1.64	0.96	0.	7.39	3.67	1.33
time (sec)	N/A	0.072	0.073	0.006	0.	1.588	2.868	1.096

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	191	232	0	2279	148	4581
normalized size	1	1.	1.1	1.33	0.	13.1	0.85	26.33
time (sec)	N/A	0.195	0.423	0.019	0.	1.625	1.808	2.354

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	135	119	0	664	345	127
normalized size	1	1.	1.52	1.34	0.	7.46	3.88	1.43
time (sec)	N/A	0.133	0.125	0.006	0.	2.017	7.246	1.095

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	151	383	0	1806	877	217
normalized size	1	1.	0.91	2.31	0.	10.88	5.28	1.31
time (sec)	N/A	0.223	0.209	0.013	0.	1.671	4.751	21.874

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	327	844	0	6311	450	0
normalized size	1	1.	0.99	2.55	0.	19.07	1.36	0.
time (sec)	N/A	0.703	0.713	0.033	0.	2.158	6.661	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	121	222	0	1412	745	205
normalized size	1	1.	0.92	1.68	0.	10.7	5.64	1.55
time (sec)	N/A	0.152	0.186	0.011	0.	1.385	3.505	25.605

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	282	602	0	4806	379	0
normalized size	1	1.	1.04	2.22	0.	17.73	1.4	0.
time (sec)	N/A	0.526	0.572	0.029	0.	1.723	4.632	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	93	104	0	864	282	130
normalized size	1	1.	1.19	1.33	0.	11.08	3.62	1.67
time (sec)	N/A	0.072	0.091	0.009	0.	1.366	1.746	29.874

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	235	452	0	3584	294	0
normalized size	1	1.	0.99	1.91	0.	15.12	1.24	0.
time (sec)	N/A	0.358	0.441	0.026	0.	1.395	3.359	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	77	0	778	267	111
normalized size	1	1.	1.05	1.03	0.	10.37	3.56	1.48
time (sec)	N/A	0.069	0.069	0.004	0.	1.282	1.667	25.713

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	222	342	0	3623	298	0
normalized size	1	1.	1.	1.55	0.	16.39	1.35	0.
time (sec)	N/A	0.24	0.462	0.063	0.	1.612	3.489	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	79	75	0	783	267	111
normalized size	1	1.	1.07	1.01	0.	10.58	3.61	1.5
time (sec)	N/A	0.065	0.085	0.004	0.	1.378	1.574	31.512

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	243	733	0	4918	394	0
normalized size	1	1.	0.96	2.91	0.	19.52	1.56	0.
time (sec)	N/A	0.461	0.461	0.044	0.	1.767	4.473	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	207	253	0	1728	772	224
normalized size	1	1.	1.7	2.07	0.	14.16	6.33	1.84
time (sec)	N/A	0.188	0.322	0.014	0.	1.784	40.968	22.145

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	302	712	0	6460	481	0
normalized size	1	1.	0.98	2.31	0.	20.97	1.56	0.
time (sec)	N/A	1.352	0.668	0.03	0.	2.108	7.145	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	248	352	0	2103	906	246
normalized size	1	1.	1.53	2.17	0.	12.98	5.59	1.52
time (sec)	N/A	0.25	0.275	0.017	0.	2.194	72.952	25.533

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	344	913	0	7960	566	0
normalized size	1	1.	0.95	2.53	0.	22.05	1.57	0.
time (sec)	N/A	3.077	0.788	0.036	0.	2.942	12.022	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	328	443	0	2627	1074	370
normalized size	1	1.	1.5	2.02	0.	12.	4.9	1.69
time (sec)	N/A	0.312	0.366	0.019	0.	2.724	131.412	23.764

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	170	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.171	0.095	0.017	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	486	1042	0	0	0	0
normalized size	1	1.	1.28	2.74	0.	0.	0.	0.
time (sec)	N/A	0.287	1.52	0.069	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	126	157	0	547	0	171
normalized size	1	1.	0.98	1.22	0.	4.24	0.	1.33
time (sec)	N/A	0.093	0.078	0.016	0.	1.397	0.	1.194

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	452	508	0	0	0	0
normalized size	1	1.	1.3	1.46	0.	0.	0.	0.
time (sec)	N/A	0.224	0.971	0.019	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	155	136	0	1575	0	0
normalized size	1	1.	0.8	0.7	0.	8.12	0.	0.
time (sec)	N/A	0.209	0.066	0.017	0.	1.641	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	192	369	0	921	0	0
normalized size	1	1.	0.79	1.51	0.	3.77	0.	0.
time (sec)	N/A	0.357	0.212	0.026	0.	1.444	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	609	1878	0	0	0	0
normalized size	1	1.	1.25	3.86	0.	0.	0.	0.
time (sec)	N/A	0.457	2.321	0.024	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	152	295	0	768	0	624
normalized size	1	1.	0.86	1.67	0.	4.34	0.	3.53
time (sec)	N/A	0.138	0.114	0.02	0.	1.511	0.	1.518

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	540	1394	0	0	0	0
normalized size	1	1.	1.27	3.28	0.	0.	0.	0.
time (sec)	N/A	0.449	1.748	0.023	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	72	0	325	0	54
normalized size	1	1.	1.	0.88	0.	3.96	0.	0.66
time (sec)	N/A	0.062	0.018	0.012	0.	1.469	0.	1.314

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	193	177	0	0	0	0
normalized size	1	1.	1.6	1.46	0.	0.	0.	0.
time (sec)	N/A	0.046	0.126	0.016	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	83	72	0	327	0	0
normalized size	1	1.	1.63	1.41	0.	6.41	0.	0.
time (sec)	N/A	0.029	0.019	0.014	0.	1.4	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	303	508	0	0	0	0
normalized size	1	1.	0.92	1.54	0.	0.	0.	0.
time (sec)	N/A	0.184	0.487	0.023	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	463	533	0	0	0	0
normalized size	1	1.	1.18	1.36	0.	0.	0.	0.
time (sec)	N/A	0.248	1.083	0.027	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	126	179	0	918	0	0
normalized size	1	1.	1.22	1.74	0.	8.91	0.	0.
time (sec)	N/A	0.073	0.083	0.019	0.	1.777	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	468	519	1136	0	0	0	0
normalized size	1	1.	1.11	2.43	0.	0.	0.	0.
time (sec)	N/A	0.409	1.351	0.029	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	160	220	0	1084	0	0
normalized size	1	1.	1.04	1.43	0.	7.04	0.	0.
time (sec)	N/A	0.175	0.082	0.019	0.	1.854	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	0	0	186	0	0
normalized size	1	1.	0.9	0.	0.	3.65	0.	0.
time (sec)	N/A	0.05	0.088	0.074	0.	1.657	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	239	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.399	5.135	0.02	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	142	0	0
normalized size	1	1.	1.62	1.29	0.	3.16	0.	0.
time (sec)	N/A	0.009	0.018	0.011	0.	1.268	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	142	0	0
normalized size	1	1.	1.62	1.29	0.	3.16	0.	0.
time (sec)	N/A	0.012	0.003	0.008	0.	1.37	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	142	0	0
normalized size	1	1.	1.62	1.29	0.	3.16	0.	0.
time (sec)	N/A	0.012	0.004	0.003	0.	1.284	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	0	157	0	93
normalized size	1	1.	0.81	0.94	0.	1.83	0.	1.08
time (sec)	N/A	0.041	0.034	0.007	0.	1.296	0.	1.089

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	0	157	0	93
normalized size	1	1.	0.81	0.94	0.	1.83	0.	1.08
time (sec)	N/A	0.042	0.009	0.003	0.	1.288	0.	1.092

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	0	157	0	93
normalized size	1	1.	0.81	0.94	0.	1.83	0.	1.08
time (sec)	N/A	0.041	0.003	0.003	0.	1.293	0.	1.096

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	35	0	273	0	47
normalized size	1	1.	0.97	0.92	0.	7.18	0.	1.24
time (sec)	N/A	0.016	0.009	0.003	0.	1.361	0.	1.097

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	70	64	0	300	0	80
normalized size	1	1.	1.56	1.42	0.	6.67	0.	1.78
time (sec)	N/A	0.022	0.023	0.008	0.	1.394	0.	1.104

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	72	64	0	315	0	47
normalized size	1	1.	1.53	1.36	0.	6.7	0.	1.
time (sec)	N/A	0.076	0.033	0.011	0.	1.704	0.	1.145

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	74	66	0	325	0	47
normalized size	1	1.	1.51	1.35	0.	6.63	0.	0.96
time (sec)	N/A	0.09	0.028	0.006	0.	1.707	0.	1.157

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	294	0	0
normalized size	1	1.	1.	0.89	0.	6.68	0.	0.
time (sec)	N/A	0.036	0.012	0.004	0.	1.467	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	81	72	0	311	0	0
normalized size	1	1.	1.65	1.47	0.	6.35	0.	0.
time (sec)	N/A	0.015	0.016	0.005	0.	1.325	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	83	72	0	327	0	0
normalized size	1	1.	1.63	1.41	0.	6.41	0.	0.
time (sec)	N/A	0.067	0.02	0.008	0.	1.372	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	85	74	0	338	0	0
normalized size	1	1.	1.6	1.4	0.	6.38	0.	0.
time (sec)	N/A	0.077	0.019	0.007	0.	1.362	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	27	131	0	0
normalized size	1	1.	1.	0.78	0.68	3.28	0.	0.
time (sec)	N/A	0.027	0.009	0.003	1.646	1.311	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	142	0	0
normalized size	1	1.	1.62	1.29	0.	3.16	0.	0.
time (sec)	N/A	0.012	0.008	0.	0.	1.289	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	62	50	0	132	0	69
normalized size	1	1.	1.44	1.16	0.	3.07	0.	1.6
time (sec)	N/A	0.047	0.02	0.011	0.	1.393	0.	1.144

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.393	0.121	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [86] had the largest ratio of [0.5]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	20	0.05
2	A	2	1	1.	18	0.056
3	A	1	0	1.	16	0.
4	A	2	1	1.	20	0.05
5	A	2	1	1.	20	0.05
6	A	3	2	1.	22	0.091
7	A	3	2	1.	20	0.1
8	A	3	2	1.	18	0.111
9	A	3	2	1.	22	0.091
10	A	3	2	1.	22	0.091
11	A	7	6	1.	22	0.273
12	A	6	6	1.	22	0.273
13	A	5	5	1.	22	0.227
14	A	3	3	1.	22	0.136
15	A	7	7	1.	20	0.35
16	A	8	7	1.	18	0.389
17	A	8	7	1.	22	0.318
18	A	8	7	1.	22	0.318
19	A	8	7	1.	22	0.318
20	A	7	7	1.	22	0.318

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	4	4	1.	22	0.182
22	A	4	4	1.	22	0.182
23	A	4	4	1.	22	0.182
24	A	8	7	1.	22	0.318
25	A	8	7	1.	22	0.318
26	A	8	7	1.	20	0.35
27	A	8	7	1.	18	0.389
28	A	8	7	1.	22	0.318
29	A	8	6	1.	24	0.25
30	A	7	6	1.	22	0.273
31	A	5	5	1.	20	0.25
32	A	4	4	1.	24	0.167
33	A	7	6	1.	24	0.25
34	A	7	6	1.	24	0.25
35	A	5	5	1.	24	0.208
36	A	6	5	1.	24	0.208
37	A	7	5	1.	24	0.208
38	A	10	7	1.	22	0.318
39	A	10	7	1.	20	0.35
40	A	8	7	1.	24	0.292
41	A	6	5	1.	24	0.208
42	A	5	4	1.	24	0.167
43	A	8	7	1.	24	0.292
44	A	8	7	1.	24	0.292
45	A	8	7	1.	24	0.292
46	A	9	8	1.	24	0.333
47	A	7	6	1.	24	0.25
48	A	8	6	1.	24	0.25
49	A	6	6	1.	24	0.25
50	A	4	4	1.	24	0.167
51	A	3	3	1.	22	0.136
52	A	2	2	1.	20	0.1
53	A	3	3	1.	24	0.125
54	A	5	5	1.	24	0.208
55	A	8	6	1.	24	0.25
56	A	7	6	1.	24	0.25
57	A	6	6	1.	24	0.25
58	A	1	1	1.	24	0.042
59	A	1	1	1.	24	0.042
60	A	3	3	1.	24	0.125
61	A	5	5	1.	22	0.227
62	A	6	5	1.	20	0.25
63	A	7	5	1.	24	0.208
64	A	8	5	1.	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
65	A	2	1	1.	18	0.056
66	A	2	1	1.	18	0.056
67	A	2	1	1.	16	0.062
68	A	1	0	1.	14	0.
69	A	2	1	1.	18	0.056
70	A	2	1	1.	18	0.056
71	A	2	1	1.	18	0.056
72	A	3	2	1.	20	0.1
73	A	3	2	1.	20	0.1
74	A	4	3	1.	18	0.167
75	A	3	2	1.	16	0.125
76	A	4	3	1.	20	0.15
77	A	3	2	1.	20	0.1
78	A	8	7	1.	20	0.35
79	A	6	5	1.	20	0.25
80	A	7	7	1.	20	0.35
81	A	5	4	1.	20	0.2
82	A	6	6	1.	20	0.3
83	A	4	3	1.	20	0.15
84	A	4	4	1.	20	0.2
85	A	4	3	1.	18	0.167
86	A	8	8	1.	16	0.5
87	A	5	4	1.	20	0.2
88	A	9	8	1.	20	0.4
89	A	9	8	1.	20	0.4
90	A	7	5	1.	20	0.25
91	A	8	8	1.	20	0.4
92	A	6	5	1.	20	0.25
93	A	5	5	1.	20	0.25
94	A	5	4	1.	20	0.2
95	A	5	5	1.	20	0.25
96	A	5	4	1.	20	0.2
97	A	5	5	1.	20	0.25
98	A	5	4	1.	20	0.2
99	A	9	8	1.	18	0.444
100	A	6	5	1.	16	0.312
101	A	9	8	1.	20	0.4
102	A	7	5	1.	20	0.25
103	A	9	8	1.	20	0.4
104	A	3	3	1.	20	0.15
105	A	5	5	1.	24	0.208
106	A	5	5	1.	24	0.208
107	A	5	5	1.	24	0.208
108	A	8	7	1.	24	0.292

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
109	A	8	8	1.	24	0.333
110	A	6	6	1.	24	0.25
111	A	6	5	1.	24	0.208
112	A	6	6	1.	24	0.25
113	A	4	4	1.	24	0.167
114	A	2	2	1.	24	0.083
115	A	2	2	1.	24	0.083
116	A	6	6	1.	24	0.25
117	A	5	5	1.	24	0.208
118	A	3	3	1.	24	0.125
119	A	6	6	1.	24	0.25
120	A	5	5	1.	24	0.208
121	A	1	1	1.	34	0.029
122	A	7	4	1.	27	0.148
123	A	2	2	1.	18	0.111
124	A	3	3	1.	18	0.167
125	A	3	3	1.	17	0.176
126	A	5	5	1.	18	0.278
127	A	6	6	1.	18	0.333
128	A	6	6	1.	17	0.353
129	A	2	2	1.	18	0.111
130	A	3	3	1.	18	0.167
131	A	3	3	1.	22	0.136
132	A	3	3	1.	24	0.125
133	A	3	3	1.	20	0.15
134	A	3	3	1.	20	0.15
135	A	3	3	1.	24	0.125
136	A	3	3	1.	26	0.115
137	A	3	3	1.	18	0.167
138	A	3	3	1.	18	0.167
139	A	3	3	1.	20	0.15
140	A	2	2	1.	36	0.056

Chapter 3

Listing of integrals

3.1 $\int x^2 \left(ax^2 + bx^3 + cx^4\right) dx$

Optimal. Leaf size=25

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

```
[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Rubi [A] time = 0.0074164, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {14}

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(ax^2 + bx^3 + cx^4\right) dx &= \int \left(ax^4 + bx^5 + cx^6\right) dx \\ &= \frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7} \end{aligned}$$

Mathematica [A] time = 0.0022189, size = 25, normalized size = 1.

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/5*a*x^5+1/6*b*x^6+1/7*c*x^7
```

Maxima [A] time = 1.09851, size = 26, normalized size = 1.04

$$\frac{1}{7}cx^7 + \frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Fricas [A] time = 1.27924, size = 47, normalized size = 1.88

$$\frac{1}{7}x^7c + \frac{1}{6}x^6b + \frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c + 1/6*x^6*b + 1/5*x^5*a
```

Sympy [A] time = 0.058303, size = 19, normalized size = 0.76

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] a*x**5/5 + b*x**6/6 + c*x**7/7
```

Giac [A] time = 1.09862, size = 26, normalized size = 1.04

$$\frac{1}{7}cx^7 + \frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

3.2 $\int x\left(ax^2+bx^3+cx^4\right)\,dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

```
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Rubi [A] time = 0.0073601, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3+cx^4\right)\,dx&=\int \left(ax^3+bx^4+cx^5\right)\,dx\\&=\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.0015328, size = 25, normalized size = 1.

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

Maxima [A] time = 1.15779, size = 26, normalized size = 1.04

$$\frac{1}{6}cx^6 + \frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Fricas [A] time = 1.30137, size = 47, normalized size = 1.88

$$\frac{1}{6}x^6c + \frac{1}{5}x^5b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c + 1/5*x^5*b + 1/4*x^4*a
```

Sympy [A] time = 0.05838, size = 19, normalized size = 0.76

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] a*x**4/4 + b*x**5/5 + c*x**6/6
```

Giac [A] time = 1.10149, size = 26, normalized size = 1.04

$$\frac{1}{6}cx^6 + \frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

3.3 $\int \left(ax^2 + bx^3 + cx^4\right) dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

```
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Rubi [A] time = 0.00384, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a*x^2 + b*x^3 + c*x^4,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Rubi steps

$$\int \left(ax^2 + bx^3 + cx^4\right) dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Mathematica [A] time = 0.0000454, size = 25, normalized size = 1.

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x^2 + b*x^3 + c*x^4,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^3+a*x^2,x)
```

```
[Out] 1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

Maxima [A] time = 1.0763, size = 26, normalized size = 1.04

$$\frac{1}{5}\,cx^5 + \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Fricas [A] time = 1.36483, size = 47, normalized size = 1.88

$$\frac{1}{5}x^5c + \frac{1}{4}x^4b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c + 1/4*x^4*b + 1/3*x^3*a
```

Sympy [A] time = 0.057533, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4 + c*x**5/5
```

Giac [A] time = 1.08135, size = 26, normalized size = 1.04

$$\frac{1}{5}\,cx^5 + \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

3.4 $\int \frac{ax^2+bx^3+cx^4}{x} \, dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

```
[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Rubi [A] time = 0.0061748, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^2+bx^3+cx^4}{x} \, dx &= \int \left(ax+bx^2+cx^3\right) \, dx\\ &= \frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}\end{aligned}$$

Mathematica [A] time = 0.0014316, size = 25, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)/x,x)
```

```
[Out] 1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

Maxima [A] time = 1.07064, size = 26, normalized size = 1.04

$$\frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Fricas [A] time = 1.44192, size = 47, normalized size = 1.88

$$\frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] time = 0.057205, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)/x,x)
```

```
[Out] a*x**2/2 + b*x**3/3 + c*x**4/4
```

Giac [A] time = 1.0827, size = 26, normalized size = 1.04

$$\frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

3.5 $\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx$

Optimal. Leaf size=20

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Rubi [A] time = 0.0063249, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {14}

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)/x^2,x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx &= \int \left(a+bx+cx^2\right)\,dx\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}\end{aligned}$$

Mathematica [A] time = 0.001181, size = 20, normalized size = 1.

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x^2,x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)/x^2,x)

[Out] a*x+1/2*b*x^2+1/3*c*x^3
```

Maxima [A] time = 1.11766, size = 22, normalized size = 1.1

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="maxima")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Fricas [A] time = 1.46768, size = 39, normalized size = 1.95

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="fricas")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [A] time = 0.056357, size = 15, normalized size = 0.75

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)/x**2,x)

[Out] a*x + b*x**2/2 + c*x**3/3
```

Giac [A] time = 1.07503, size = 22, normalized size = 1.1

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="giac")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.6 $\int x^2 \left(ax^2+bx^3+cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^1
1)/11
```

Rubi [A] time = 0.053594, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1585, 698}

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^1
1)/11
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx &= \int x^6\left(a+bx+cx^2\right)^2\,dx\\ &= \int \left(a^2x^6+2abx^7+\left(b^2+2ac\right)x^8+2bcx^9+c^2x^{10}\right)dx\\ &= \frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.0101222, size = 54, normalized size = 1.

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^1
1)/11
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{\left(2\,ac+b^2\right)x^9}{9}+\frac{bcx^{10}}{5}+\frac{c^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

Maxima [A] time = 1.16539, size = 59, normalized size = 1.09

$$\frac{1}{11}\,c^2x^{11}+\frac{1}{5}\,bcx^{10}+\frac{1}{4}\,abx^8+\frac{1}{9}\left(b^2+2\,ac\right)x^9+\frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a^
2*x^7
```

Fricas [A] time = 1.26837, size = 116, normalized size = 2.15

$$\frac{1}{11}x^{11}c^2+\frac{1}{5}x^{10}cb+\frac{1}{9}x^9b^2+\frac{2}{9}x^9ca+\frac{1}{4}x^8ba+\frac{1}{7}x^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2 + 1/5*x^10*c*b + 1/9*x^9*b^2 + 2/9*x^9*c*a + 1/4*x^8*b*a + 1/
7*x^7*a^2
```

Sympy [A] time = 0.070734, size = 48, normalized size = 0.89

$$\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{bcx^{10}}{5}+\frac{c^2x^{11}}{11}+x^9\left(\frac{2ac}{9}+\frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**7/7 + a*b*x**8/4 + b*c*x**10/5 + c**2*x**11/11 + x**9*(2*a*c/9 + b*
*2/9)
```

Giac [A] time = 1.07082, size = 62, normalized size = 1.15

$$\frac{1}{11}c^2x^{11} + \frac{1}{5}bcx^{10} + \frac{1}{9}b^2x^9 + \frac{2}{9}acx^9 + \frac{1}{4}abx^8 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 1/4*a*b*x^8 + 1/
7*a^2*x^7
```

3.7 $\int x\left(ax^2+bx^3+cx^4\right)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*
x^10)/10
```

Rubi [A] time = 0.028877, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1585, 698}

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*
x^10)/10
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3+cx^4\right)^2 dx &= \int x^5\left(a+bx+cx^2\right)^2 dx\\ &= \int \left(a^2x^5+2abx^6+\left(b^2+2ac\right)x^7+2bcx^8+c^2x^9\right) dx\\ &= \frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.0068741, size = 54, normalized size = 1.

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*
x^10)/10
```

Maple [A] time = 0., size = 45, normalized size = 0.8

$$\frac{a^2x^6}{6}+\frac{2\,abx^7}{7}+\frac{\left(2\,ac+b^2\right)x^8}{8}+\frac{2\,bcx^9}{9}+\frac{c^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

Maxima [A] time = 1.12733, size = 59, normalized size = 1.09

$$\frac{1}{10}\,c^2x^{10}+\frac{2}{9}\,bcx^9+\frac{2}{7}\,abx^7+\frac{1}{8}\left(b^2+2\,ac\right)x^8+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^2
*x^6
```

Fricas [A] time = 1.28945, size = 115, normalized size = 2.13

$$\frac{1}{10}x^{10}c^2+\frac{2}{9}x^9cb+\frac{1}{8}x^8b^2+\frac{1}{4}x^8ca+\frac{2}{7}x^7ba+\frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^2 + 2/9*x^9*c*b + 1/8*x^8*b^2 + 1/4*x^8*c*a + 2/7*x^7*b*a + 1/6
*x^6*a^2
```

Sympy [A] time = 0.069998, size = 49, normalized size = 0.91

$$\frac{a^2x^6}{6}+\frac{2abx^7}{7}+\frac{2bcx^9}{9}+\frac{c^2x^{10}}{10}+x^8\left(\frac{ac}{4}+\frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + 2*b*c*x**9/9 + c**2*x**10/10 + x**8*(a*c/4 + b
**2/8)
```

Giac [A] time = 1.08124, size = 62, normalized size = 1.15

$$\frac{1}{10}c^2x^{10} + \frac{2}{9}bcx^9 + \frac{1}{8}b^2x^8 + \frac{1}{4}acx^8 + \frac{2}{7}abx^7 + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 2/7*a*b*x^7 + 1/6
*a^2*x^6
```

3.8 $\int \left(ax^2+bx^3+cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)
/9
```

Rubi [A] time = 0.0265261, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1594, 698}

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)
/9
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}
\int \left(ax^2+bx^3+cx^4\right)^2 \, dx &= \int x^4\left(a+bx+cx^2\right)^2 \, dx\\
&= \int \left(a^2x^4+2abx^5+\left(b^2+2ac\right)x^6+2bcx^7+c^2x^8\right) dx\\
&= \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}
\end{aligned}$$

Mathematica [A] time = 0.0060407, size = 54, normalized size = 1.

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)
/9
```

Maple [A] time = 0., size = 45, normalized size = 0.8

$$\frac{a^2x^5}{5}+\frac{abx^6}{3}+\frac{\left(2\,ac+b^2\right)x^7}{7}+\frac{bcx^8}{4}+\frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

Maxima [A] time = 1.33673, size = 65, normalized size = 1.2

$$\frac{1}{9}\,c^2x^9+\frac{1}{4}\,bcx^8+\frac{1}{7}\,b^2x^7+\frac{1}{5}\,a^2x^5+\frac{1}{21}\left(6\,cx^7+7\,bx^6\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 1/5*a^2*x^5 + 1/21*(6*c*x^7 + 7*b
*x^6)*a
```

Fricas [A] time = 1.36451, size = 112, normalized size = 2.07

$$\frac{1}{9}x^9c^2+\frac{1}{4}x^8cb+\frac{1}{7}x^7b^2+\frac{2}{7}x^7ca+\frac{1}{3}x^6ba+\frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 1/4*x^8*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 1/3*x^6*b*a + 1/5*x
^5*a^2
```

Sympy [A] time = 0.073278, size = 48, normalized size = 0.89

$$\frac{a^2x^5}{5}+\frac{abx^6}{3}+\frac{bcx^8}{4}+\frac{c^2x^9}{9}+x^7\left(\frac{2ac}{7}+\frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**5/5 + a*b*x**6/3 + b*c*x**8/4 + c**2*x**9/9 + x**7*(2*a*c/7 + b**2/
7)
```

Giac [A] time = 1.09585, size = 62, normalized size = 1.15

$$\frac{1}{9}c^2x^9 + \frac{1}{4}bcx^8 + \frac{1}{7}b^2x^7 + \frac{2}{7}acx^7 + \frac{1}{3}abx^6 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 1/3*a*b*x^6 + 1/5*a
^2*x^5
```

3.9 $\int \frac{(ax^2+bx^3+cx^4)^2}{x}\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*
x^8)/8
```

Rubi [A] time = 0.0320555, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1585, 698}

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*
x^8)/8
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x}\,dx &= \int x^3\left(a+bx+cx^2\right)^2\,dx\\ &= \int \left(a^2x^3+2abx^4+\left(b^2+2ac\right)x^5+2bcx^6+c^2x^7\right)dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.0062127, size = 54, normalized size = 1.

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*
x^8)/8
```

Maple [A] time = 0.002, size = 45, normalized size = 0.8

$$\frac{a^2x^4}{4}+\frac{2\,abx^5}{5}+\frac{\left(2\,ac+b^2\right)x^6}{6}+\frac{2\,bcx^7}{7}+\frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^2/x,x)
```

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8
```

Maxima [A] time = 1.12473, size = 59, normalized size = 1.09

$$\frac{1}{8}\,c^2x^8+\frac{2}{7}\,bcx^7+\frac{2}{5}\,abx^5+\frac{1}{6}\left(b^2+2\,ac\right)x^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*x
^4
```

Fricas [A] time = 1.42506, size = 107, normalized size = 1.98

$$\frac{1}{8}\,c^2x^8+\frac{2}{7}\,bcx^7+\frac{2}{5}\,abx^5+\frac{1}{6}\left(b^2+2\,ac\right)x^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*x
^4
```

Sympy [A] time = 0.070307, size = 49, normalized size = 0.91

$$\frac{a^2x^4}{4}+\frac{2abx^5}{5}+\frac{2bcx^7}{7}+\frac{c^2x^8}{8}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2/x,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + 2*b*c*x**7/7 + c**2*x**8/8 + x**6*(a*c/3 + b**
2/6)
```

Giac [A] time = 1.09907, size = 62, normalized size = 1.15

$$\frac{1}{8}c^2x^8 + \frac{2}{7}bcx^7 + \frac{1}{6}b^2x^6 + \frac{1}{3}acx^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 2/5*a*b*x^5 + 1/4*a
^2*x^4
```

3.10 $\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3}+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)
/7
```

Rubi [A] time = 0.0324322, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1585, 698}

$$\frac{a^2x^3}{3}+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)
/7
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x^2}\,dx &= \int x^2\left(a+bx+cx^2\right)^2\,dx\\ &= \int \left(a^2x^2+2abx^3+\left(b^2+2ac\right)x^4+2bcx^5+c^2x^6\right)dx\\ &= \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0071731, size = 54, normalized size = 1.

$$\frac{a^2x^3}{3}+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)
/7
```

Maple [A] time = 0., size = 45, normalized size = 0.8

$$\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{\left(2\,ac+b^2\right)x^5}{5}+\frac{bcx^6}{3}+\frac{c^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^2/x^2,x)
```

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

Maxima [A] time = 1.15267, size = 59, normalized size = 1.09

$$\frac{1}{7}\,c^2x^7+\frac{1}{3}\,bcx^6+\frac{1}{2}\,abx^4+\frac{1}{5}\,\left(b^2+2\,ac\right)x^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*x
^3
```

Fricas [A] time = 1.45505, size = 107, normalized size = 1.98

$$\frac{1}{7}\,c^2x^7+\frac{1}{3}\,bcx^6+\frac{1}{2}\,abx^4+\frac{1}{5}\,\left(b^2+2\,ac\right)x^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*x
^3
```

Sympy [A] time = 0.072056, size = 48, normalized size = 0.89

$$\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{bcx^6}{3}+\frac{c^2x^7}{7}+x^5\left(\frac{2ac}{5}+\frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2/x**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c/5 + b**2/
5)
```

Giac [A] time = 1.07228, size = 62, normalized size = 1.15

$$\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 1/2*a*b*x^4 + 1/3*a
^2*x^3
```

3.11 $\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=89

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rubi [A] time = 0.093805, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1585, 701, 634, 618, 206, 628}

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx &= \int \frac{x^3}{a+bx+cx^2}\,dx \\
&= \int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{c^2} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^3}+\frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^3} \\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}
\end{aligned}$$

Mathematica [A] time = 0.109711, size = 84, normalized size = 0.94

$$\frac{\left(b^2-ac\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx(cx-2b)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

Maple [A] time = 0.003, size = 132, normalized size = 1.5

$$\frac{x^2}{2\,c}-\frac{bx}{c^2}-\frac{\ln\left(cx^2+bx+a\right)a}{2\,c^2}+\frac{\ln\left(cx^2+bx+a\right)b^2}{2\,c^3}+3\,\frac{ab}{c^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-\frac{b^3}{c^3}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/2*x^2/c-b*x/c^2-1/2/c^2*ln(c*x^2+b*x+a)*a+1/2/c^3*ln(c*x^2+b*x+a)*b^2+3/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b-1/c^3/(4*a*c-b
```

```
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59039, size = 653, normalized size = 7.34

$$\left[\frac{\left(b^2c^2 - 4\,ac^3\right)x^2 - \left(b^3 - 3\,abc\right)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) - 2\left(b^3c - 4\,abc^2\right)x + \left(b^4 - 5\,ab^2c + 4\right.}{2\left(b^2c^3 - 4\,ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2
+ b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*
a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 +
b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] time = 0.783942, size = 381, normalized size = 4.28

$$-\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{2c^3\left(4ac-b^2\right)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c - ab^2 + 4ac^3\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{2c^3(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c^3(4a}\right.}{3abc - b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
- (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
 - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*
c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*
c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
- b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c
 - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)
```

```
) + x**2/(2*c)
```

Giac [A] time = 1.08281, size = 116, normalized size = 1.3

$$\frac{cx^2 - 2\,bx}{2\,c^2} + \frac{\left(b^2 - ac\right)\log\left(cx^2 + bx + a\right)}{2\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 -
 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.12 $\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=70

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.0587906, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1585, 703, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx &= \int \frac{x^2}{a+bx+cx^2}\,dx \\
&= \frac{x}{c}+\frac{\int \frac{-a-bx}{a+bx+cx^2}\,dx}{c} \\
&= \frac{x}{c}-\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{\left(b^2-2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2} \\
&= \frac{x}{c}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(b^2-2ac\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2} \\
&= \frac{x}{c}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.0636759, size = 73, normalized size = 1.04

$$\frac{\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2
 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A] time = 0.003, size = 101, normalized size = 1.4

$$\frac{x}{c}-\frac{b\ln\left(cx^2+bx+a\right)}{2\,c^2}-2\,\frac{a}{c\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+\frac{b^2}{c^2}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*a+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*
b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6885, size = 537, normalized size = 7.67

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^2c-4\,ac^2\right)x+\left(b^3-4\,abc\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c^2-4\,ac^3\right)},-\frac{2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2
)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b
^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(
b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 0.647319, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*
log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c*
*2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)
)) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*
c - b**2))))/(2*a*c - b**2)) + x/c
```

Giac [A] time = 1.08301, size = 90, normalized size = 1.29

$$\frac{x}{c}-\frac{b\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.13 $\int \frac{x^3}{ax^2+bx^3+cx^4} dx$

Optimal. Leaf size=56

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rubi [A] time = 0.0395897, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1585, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx &= \int \frac{x}{a+bx+cx^2}\,dx\\ &= \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c} - \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2c}\\ &= \frac{\log\left(a+bx+cx^2\right)}{2c} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c}\\ &= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx+cx^2\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.0318506, size = 57, normalized size = 1.02

$$\frac{\log(a+x(b+cx)) - \frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a +
 x*(b + c*x)])/(2*c)
```

Maple [A] time = 0.001, size = 56, normalized size = 1.

$$\frac{\ln\left(cx^2+bx+a\right)}{2\,c} - \frac{b}{c}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/2*ln(c*x^2+b*x+a)/c-1/c*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55918, size = 427, normalized size = 7.62

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c-4\,ac^2\right)}, \frac{2\sqrt{-b^2+4\,ac}b\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\ldots}{b^2-4\ldots}\right)\ldots}{2\left(b\ldots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c
 - 4*a*c^2)]
```

Sympy [B] time = 0.301536, size = 216, normalized size = 3.86

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log\left(x+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b
*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(
-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*
c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*
c - b**2)) + 1/(2*c)))/b)
```

Giac [A] time = 1.10495, size = 74, normalized size = 1.32

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c}+\frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(
c*x^2 + b*x + a)/c
```

3.14 $\int \frac{x^2}{ax^2+bx^3+cx^4} dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.0259065, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1585, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{ax^2+bx^3+cx^4} dx &= \int \frac{1}{a+bx+cx^2} dx \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+2cx\right)\right) \\
&= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A] time = 0.0063149, size = 38, normalized size = 1.12

$$\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.002, size = 35, normalized size = 1.

$$2\frac{1}{\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57138, size = 277, normalized size = 8.15

$$\left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] time = 0.201467, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Giac [A] time = 1.11321, size = 46, normalized size = 1.35

$$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.15 $\int \frac{x}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=62

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rubi [A] time = 0.0469107, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1585, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{ax^2+bx^3+cx^4}\,dx &= \int \frac{1}{x\left(a+bx+cx^2\right)}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{a}+\frac{\int \frac{-b-cx}{a+bx+cx^2}\,dx}{a}\\
&= \frac{\log(x)}{a}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.0729725, size = 61, normalized size = 0.98

$$
-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\log(a+x(b+cx))-2\log(x)}{2a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] -((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*Log[x
] + Log[a + x*(b + c*x)])/(2*a)
```

Maple [A] time = 0.006, size = 62, normalized size = 1.

$$
\frac{\ln(x)}{a}-\frac{\ln\left(cx^2+bx+a\right)}{2\,a}-\frac{b}{a}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a-1/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.76169, size = 494, normalized size = 7.97

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)+2\left(b^2-4\,ac\right)\log\left(x\right)}{2\left(ab^2-4\,a^2c\right)},\frac{2\sqrt{-b^2+4}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*
b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log
(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] time = 1.92687, size = 564, normalized size = 9.1

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+}{9abc^2-2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**
2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a
*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4
```

```
*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
 b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

Giac [A] time = 1.08312, size = 84, normalized size = 1.35

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a}-\frac{\log\left(cx^2+bx+a\right)}{2\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(
c*x^2 + b*x + a)/a + log(abs(x))/a
```

3.16 $\int \frac{1}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rubi [A] time = 0.0976829, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1594, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax^2+bx^3+cx^4}\,dx &= \int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx \\
&= -\frac{1}{ax}+\frac{\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx}{a} \\
&= -\frac{1}{ax}+\frac{\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx}{a} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx}{a^2} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2}+\frac{\left(b^2-2ac\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2} \\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a^2} \\
&= -\frac{1}{ax}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0789887, size = 77, normalized size = 0.95

$$\frac{\frac{2\left(b^2-2ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log(a+x(b+cx))-\frac{2a}{x}-2b\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

```
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-
b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [A] time = 0.006, size = 112, normalized size = 1.4

$$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(cx^2+bx+a\right)}{2a^2}-2\frac{c}{a\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)+\frac{b^2}{a^2}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-2/a/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*c+1/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77597, size = 626, normalized size = 7.73

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-\left(b^3-4\,abc\right)x\log\left(cx^2+bx+a\right)+2\left(\right.}{2\left(a^2b^2-4\,a^3c\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*
c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((
a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4
*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*
a^3*c)*x)]
```

Sympy [B] time = 3.50723, size = 862, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))*l
og(x + (-28*a**6*b*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2
*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) - sqrt(-4*a*c + b**2
)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c**3*(b/(2*a**2) - sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**5*(b/
(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 -
```

```
 3*a**4*b**2*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*
(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b
**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2 - 14*a*b*
*5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2 + 2*b**6*c
)) + (b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2
)))*log(x + (-28*a**6*b*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(2*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) + sqrt(-4*a*c +
 b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c**3*(b/(2*a**2)
 + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**
5*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))
**2 - 3*a**4*b**2*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*
a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*
c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2 - 14
*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2 + 2*b
**6*c)) - 1/(a*x) - b*log(x)/a**2
```

Giac [A] time = 1.09282, size = 107, normalized size = 1.32

$$\frac{b\log\left(cx^2+bx+a\right)}{2\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

3.17 $\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}\,dx$

Optimal. Leaf size=104

$$-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*
Log[a + b*x + c*x^2])/(2*a^3)
```

Rubi [A] time = 0.145913, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*
Log[a + b*x + c*x^2])/(2*a^3)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}\,dx = \int \frac{1}{x^3\left(a+bx+cx^2\right)}\,dx$$
$$= -\frac{1}{2ax^2} + \frac{\int \frac{-b-cx}{x^2\left(a+bx+cx^2\right)}\,dx}{a}$$
$$= -\frac{1}{2ax^2} + \frac{\int \left(-\frac{b}{ax^2} + \frac{b^2-ac}{a^2x} + \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a^2\left(a+bx+cx^2\right)}\right)dx}{a}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{\left(b^2-ac\right)\log(x)}{a^3} + \frac{\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{a^3}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{\left(b^2-ac\right)\log(x)}{a^3} - \frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^3} - \frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^3}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{\left(b^2-ac\right)\log(x)}{a^3} - \frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3} + \frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{}{b^2}\right.}{a^3}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{\left(b^2-ac\right)\log(x)}{a^3} - \frac{\left(b^2-ac\right)\log\left(a+bx+c\right.}{2a^3}$$

Mathematica [A] time = 0.137503, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2} + 2\log(x)\left(b^2-ac\right) + \left(ac-b^2\right)\log(a+x(b+cx)) - \frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{2ab}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a +
x*(b + c*x)])/(2*a^3)
```

Maple [A] time = 0.007, size = 150, normalized size = 1.4

$$-\frac{1}{2\,a{x}^{2}}-\frac{\ln \left(x \right) c}{{a}^{2}}+\frac{\ln \left(x \right) {b}^{2}}{{a}^{3}}+\frac{b}{x{a}^{2}}+\frac{c\ln \left(c{x}^{2}+bx+a \right)}{2\,{a}^{2}}-\frac{\ln \left(c{x}^{2}+bx+a \right) {b}^{2}}{2\,{a}^{3}}+3\,\frac{bc}{{a}^{2}\sqrt{4\,ac-{b}^{2}}}\arctan \left(\frac{2\,cx+b}{\sqrt{4\,ac-{b}^{2}}} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] -1/2/a/x^2-1/a^2*ln(x)*c+1/a^3*ln(x)*b^2+b/a^2/x+1/2/a^2*c*ln(c*x^2+b*x+a)-
1/2/a^3*ln(c*x^2+b*x+a)*b^2+3/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b*c-1/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94557, size = 798, normalized size = 7.67

$$\left[-\frac{\left(b^{3}-3\,abc\right)\sqrt{b^{2}-4\,ac}x^{2}\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}(2\,cx+b)}{cx^{2}+bx+a}\right)+a^{2}b^{2}-4\,a^{3}c+\left(b^{4}-5\,ab^{2}c+4\,a^{2}c^{2}\right)x^{2}\log\left(cx^{2}+b\right.}{2\left(a^{3}b^{2}-4\,a^{4}c\right)x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*
a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4 - 5
*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a
^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^
2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

Sympy [B] time = 5.17258, size = 1525, normalized size = 14.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**
2)/(2*a**3))*log(x + (24*a**9*c**3*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(
2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 - 42*a**8*b**2*c**2*(-b*
sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(
2*a**3))**2 + 17*a**7*b**4*c*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3
*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 + 12*a**7*c**4*(-b*sqrt(-4*a*c
 + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) -
2*a**6*b**6*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2))
+ (a*c - b**2)/(2*a**3))**2 - 15*a**6*b**2*c**3*(-b*sqrt(-4*a*c + b**2)*(3*
a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 7*a**5*b**4*
c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c
- b**2)/(2*a**3)) - 12*a**5*c**5 - a**4*b**6*c*(-b*sqrt(-4*a*c + b**2)*(3*a
*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 63*a**4*b**2*
c**4 - 103*a**3*b**4*c**3 + 70*a**2*b**6*c**2 - 20*a*b**8*c + 2*b**10)/(27*
a**4*b*c**5 - 63*a**3*b**3*c**4 + 54*a**2*b**5*c**3 - 18*a*b**7*c**2 + 2*b*
*9*c)) + (b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a
*c - b**2)/(2*a**3))*log(x + (24*a**9*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c -
b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 - 42*a**8*b**2*c*
*2*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b
**2)/(2*a**3))**2 + 17*a**7*b**4*c*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2
*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 + 12*a**7*c**4*(b*sqrt(-4
*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)
) - 2*a**6*b**6*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2
)) + (a*c - b**2)/(2*a**3))**2 - 15*a**6*b**2*c**3*(b*sqrt(-4*a*c + b**2)*(
3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 7*a**5*b**
4*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c
 - b**2)/(2*a**3)) - 12*a**5*c**5 - a**4*b**6*c*(b*sqrt(-4*a*c + b**2)*(3*a
*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 63*a**4*b**2*
c**4 - 103*a**3*b**4*c**3 + 70*a**2*b**6*c**2 - 20*a*b**8*c + 2*b**10)/(27*
a**4*b*c**5 - 63*a**3*b**3*c**4 + 54*a**2*b**5*c**3 - 18*a*b**7*c**2 + 2*b*
*9*c)) + (-a + 2*b*x)/(2*a**2*x**2) - (a*c - b**2)*log(x + (-12*a**5*c**5 +
 63*a**4*b**2*c**4 - 12*a**4*c**4*(a*c - b**2) - 103*a**3*b**4*c**3 + 15*a*
*3*b**2*c**3*(a*c - b**2) + 24*a**3*c**3*(a*c - b**2)**2 + 70*a**2*b**6*c**
2 - 7*a**2*b**4*c**2*(a*c - b**2) - 42*a**2*b**2*c**2*(a*c - b**2)**2 - 20*
a*b**8*c + a*b**6*c*(a*c - b**2) + 17*a*b**4*c*(a*c - b**2)**2 + 2*b**10 -
2*b**6*(a*c - b**2)**2)/(27*a**4*b*c**5 - 63*a**3*b**3*c**4 + 54*a**2*b**5*
c**3 - 18*a*b**7*c**2 + 2*b**9*c))/a**3
```

Giac [A] time = 1.13193, size = 142, normalized size = 1.37

$$-\frac{\left(b^2-ac\right)\log\left(cx^2+bx+a\right)}{2\,a^3}+\frac{\left(b^2-ac\right)\log\left(|x|\right)}{a^3}-\frac{\left(b^3-3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (
b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

3.18 $\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx$

Optimal. Leaf size=137

$$-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}+\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}-\frac{b^2-ac}{a^3x}-\frac{b\log(x)(b^2-2ac)}{a^4}+\frac{b}{2a^2x^2}-$$

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) -
(b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^
4)
```

Rubi [A] time = 0.202787, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 709, 800, 634, 618, 206, 628}

$$-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}+\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}-\frac{b^2-ac}{a^3x}-\frac{b\log(x)(b^2-2ac)}{a^4}+\frac{b}{2a^2x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) -
(b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^
4)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}\,dx = \int \frac{1}{x^4\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{1}{3ax^3} + \frac{\int \frac{-b-cx}{x^3\left(a+bx+cx^2\right)}\,dx}{a}$$

$$= -\frac{1}{3ax^3} + \frac{\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a^3\left(a+bx+cx^2\right)}\right) dx}{a}$$

$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4} + \frac{\int \frac{b^4-3ab^2c+a^2c^2+bc\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{a^4}$$

$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4} + \frac{\left(b\left(b^2-2ac\right)\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4} + \frac{\left(b^4-4ab^2\right.}{}$$

$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4} + \frac{b\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4} - \frac{\left(b^4-4\right.}{}$$

$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{\left(b^4-4ab^2c+2a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{b\left(b^2-2ac\right)\log(x)}{a^4} +$$

Mathematica [A] time = 0.114359, size = 131, normalized size = 0.96

$$\frac{\frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{3a^2b}{x^2} - \frac{2a^3}{x^3} + \frac{6a\left(ac-b^2\right)}{x} - 6\log(x)\left(b^3-2abc\right) + 3\left(b^3-2abc\right)\log(a+x(b+cx))}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6
*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4)
```

Maple [A] time = 0.009, size = 214, normalized size = 1.6

$$-\frac{1}{3\,ax^{3}}+\frac{c}{xa^{2}}-\frac{b^{2}}{a^{3}x}+2\,\frac{b\ln\left(x\right)c}{a^{3}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b}{2\,a^{2}x^{2}}-\frac{c\ln\left(cx^{2}+bx+a\right)b}{a^{3}}+\frac{\ln\left(cx^{2}+bx+a\right)b^{3}}{2\,a^{4}}+2\,\frac{c^{2}}{a^{2}\sqrt{4\,ac-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] -1/3/a/x^3+1/a^2/x*c-1/a^3/x*b^2+2*b/a^3*ln(x)*c-b^3/a^4*ln(x)+1/2*b/a^2/x^
2-1/a^3*c*ln(c*x^2+b*x+a)*b+1/2/a^4*ln(c*x^2+b*x+a)*b^3+2/a^2/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2-4/a^3/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c+1/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03884, size = 979, normalized size = 7.15

$$\left[\frac{3\left(b^{4}-4\,ab^{2}c+2\,a^{2}c^{2}\right)\sqrt{b^{2}-4\,ac}x^{3}\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}(2\,cx+b)}{cx^{2}+bx+a}\right)-2\,a^{3}b^{2}+8\,a^{4}c+3\left(b^{5}-6\,ab^{3}c+8\,a^{2}bc^{2}\right)}{6\left(a^{4}b^{2}-4\,a^{5}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
 - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 +
b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*
b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^
3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c +
8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*
b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [B] time = 8.80404, size = 2105, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c
 + b**4)/(2*a**4*(4*a*c - b**2)))*log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b*
*2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**
4*(4*a*c - b**2)))**2 + 57*a**10*b**3*c**2*(-b*(2*a*c - b**2)/(2*a**4) - sq
rt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))
)**2 - 19*a**9*b**5*c*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*
a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 + 4*a**9*c**5*(-
b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c +
 b**4)/(2*a**4*(4*a*c - b**2))) + 2*a**8*b**7*(-b*(2*a*c - b**2)/(2*a**4) -
 sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**
2)))**2 + 23*a**8*b**2*c**4*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**
2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) - 26*a**7*b**
4*c**3*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a
*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 9*a**6*b**6*c**2*(-b*(2*a*c - b*
*2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**
4*(4*a*c - b**2))) - 8*a**6*b*c**6 - a**5*b**8*c*(-b*(2*a*c - b**2)/(2*a**4
) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c -
b**2))) + 166*a**5*b**3*c**5 - 361*a**4*b**5*c**4 + 312*a**3*b**7*c**3 - 13
0*a**2*b**9*c**2 + 26*a*b**11*c - 2*b**13)/(2*a**6*c**7 + 60*a**5*b**2*c**6
 - 207*a**4*b**4*c**5 + 224*a**3*b**6*c**4 - 108*a**2*b**8*c**3 + 24*a*b**1
0*c**2 - 2*b**12*c)) + (-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2
*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*log(x + (-52*a**11
*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 + 57*a**10*b**3*c**2*(-b*(2*a*
c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/
(2*a**4*(4*a*c - b**2)))**2 - 19*a**9*b**5*c*(-b*(2*a*c - b**2)/(2*a**4) +
sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2
)))**2 + 4*a**9*c**5*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a
**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 2*a**8*b**7*(-b*(2
*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**
4)/(2*a**4*(4*a*c - b**2)))**2 + 23*a**8*b**2*c**4*(-b*(2*a*c - b**2)/(2*a*
*4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c
- b**2))) - 26*a**7*b**4*c**3*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b
**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 9*a**6*b*
*6*c**2*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) - 8*a**6*b*c**6 - a**5*b**8*c*(-b
*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c +
b**4)/(2*a**4*(4*a*c - b**2))) + 166*a**5*b**3*c**5 - 361*a**4*b**5*c**4 +
312*a**3*b**7*c**3 - 130*a**2*b**9*c**2 + 26*a*b**11*c - 2*b**13)/(2*a**6*c
**7 + 60*a**5*b**2*c**6 - 207*a**4*b**4*c**5 + 224*a**3*b**6*c**4 - 108*a**
2*b**8*c**3 + 24*a*b**10*c**2 - 2*b**12*c)) + (-2*a**2 + 3*a*b*x + x**2*(6*
a*c - 6*b**2))/(6*a**3*x**3) + b*(2*a*c - b**2)*log(x + (-8*a**6*b*c**6 + 1
66*a**5*b**3*c**5 + 4*a**5*b*c**5*(2*a*c - b**2) - 361*a**4*b**5*c**4 + 23*
a**4*b**3*c**4*(2*a*c - b**2) + 312*a**3*b**7*c**3 - 26*a**3*b**5*c**3*(2*a
*c - b**2) - 52*a**3*b**3*c**3*(2*a*c - b**2)**2 - 130*a**2*b**9*c**2 + 9*a
**2*b**7*c**2*(2*a*c - b**2) + 57*a**2*b**5*c**2*(2*a*c - b**2)**2 + 26*a*b
**11*c - a*b**9*c*(2*a*c - b**2) - 19*a*b**7*c*(2*a*c - b**2)**2 - 2*b**13
+ 2*b**9*(2*a*c - b**2)**2)/(2*a**6*c**7 + 60*a**5*b**2*c**6 - 207*a**4*b**
4*c**5 + 224*a**3*b**6*c**4 - 108*a**2*b**8*c**3 + 24*a*b**10*c**2 - 2*b**1
2*c))/a**4
```

Giac [A] time = 1.09323, size = 184, normalized size = 1.34

$$\frac{\left(b^3-2\,abc\right)\log\left(cx^2+bx+a\right)}{2\,a^4}-\frac{\left(b^3-2\,abc\right)\log\left(|x|\right)}{a^4}+\frac{\left(b^4-4\,ab^2c+2\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^4}+\frac{3\,a^2bx-2\,a^3-6}{6\,a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/
a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/
(a^4*x^3)
```

3.19 $\int \frac{x^8}{\left(ax^2+bx^3+cx^4\right)^2} \, dx$

Optimal. Leaf size=150

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}-\frac{b\log\left(a+bx+c\right.}{c^3}$$

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*
(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^
2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) -
(b*Log[a + b*x + c*x^2])/c^3
```

Rubi [A] time = 0.159277, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 738, 800, 634, 618, 206, 628}

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}-\frac{b\log\left(a+bx+c\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```


```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*
(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^
2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) -
(b*Log[a + b*x + c*x^2])/c^3
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^8}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{x^4}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \left(-\frac{2\left(b^2-3ac\right)}{c^2}+\frac{2bx}{c}+\frac{2\left(a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x\right)}{c^2\left(a+bx+cx^2\right)}\right)dx}{-b^2+4ac}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2\int \frac{a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{c^2\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{c^3} + \frac{\left(b^4-6ab^2c \ldots\right.}{c \ldots}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{b\log\left(a+bx+cx^2\right)}{c^3} - \frac{\left(2\left(b^4 \ldots\right.\right.}{\ldots}$$

$$= \frac{2\left(b^2-3ac\right)x}{c^2\left(b^2-4ac\right)} - \frac{bx^2}{c\left(b^2-4ac\right)} + \frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\ldots}{c^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.192539, size = 132, normalized size = 0.88

$$\frac{-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{a^2c(3b-2cx)-ab^2(b-4cx)+b^4(-x)}{\left(b^2-4ac\right)(a+x(b+cx))}-b\log(a+x(b+cx))+cx}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

Maple [B] time = 0.008, size = 352, normalized size = 2.4

$$\frac{x}{c^2}+2\,\frac{xa^2}{c\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-4\,\frac{axb^2}{c^2\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}+\frac{xb^4}{c^3\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-3\,\frac{a^2b}{c^2\left(cx^2+bx+a\right)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] x/c^2+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x
*a*b^2+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4-3/c^2/(c*x^2+b*x+a)*b*a^2/(4*a
*c-b^2)+1/c^3/(c*x^2+b*x+a)*b^3*a/(4*a*c-b^2)-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*
x+a)*a*b+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3-12/c/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2+12/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*a*b^2-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59795, size = 1760, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^
2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5
 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 -
```

```
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] time = 1.62455, size = 842, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*
(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**
4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2
*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4
*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4)
 + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

Giac [A] time = 1.11596, size = 217, normalized size = 1.45

$$\frac{2\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^3-4\,ac^4\right)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2}-\frac{b\log\left(cx^2+bx+a\right)}{c^3}-\frac{\frac{\left(b^4-4\,ab^2c+2\,a^2c^2\right)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3 -
 ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x
+ a)*(b^2 - 4*a*c)*c^2)
```

3.20 $\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal. Leaf size=114

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rubi [A] time = 0.103282, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 738, 773, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^2}\,dx &= \int \frac{x^3}{\left(a+bx+cx^2\right)^2}\,dx \\
&= \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx}{-b^2+4ac} \\
&= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{-ab+\left(-b^2+4ac\right)x}{a+bx+cx^2}\,dx}{c\left(b^2-4ac\right)} \\
&= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} - \frac{\left(b\left(b^2-6ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2\left(b^2-4ac\right)} \\
&= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log\left(a+bx+cx^2\right)}{2c^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\int \right.}{c^2\left(b^2\right.} \\
&= -\frac{bx}{c\left(b^2-4ac\right)} + \frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}} + \frac{\log\left(a+bx\right.}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.153361, size = 109, normalized size = 0.96

$$\frac{\frac{2\left(-2a^2c+ab(b-3cx)+b^3x\right)}{\left(b^2-4ac\right)(a+x(b+cx))} + \frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
```

```
)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [A] time = 0.009, size = 209, normalized size = 1.8

$$\frac{1}{cx^2+bx+a}\left(\frac{b\left(3ac-b^2\right)x}{c^2\left(4ac-b^2\right)}+\frac{a\left(2ac-b^2\right)}{c^2\left(4ac-b^2\right)}\right)+2\frac{\ln\left(cx^2+bx+a\right)a}{c\left(4ac-b^2\right)}-\frac{\ln\left(cx^2+bx+a\right)b^2}{2c^2\left(4ac-b^2\right)}-6\frac{ab}{c\left(4ac-b^2\right)^{3/2}}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+
a)+2/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^
2-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b+1/c^2/(4*a*
c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.65751, size = 1374, normalized size = 12.05

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+\left(ab^3-6\,a^2bc+\left(b^3c-6\,abc^2\right)x^2+\left(b^4-6\,ab^2c\right)x\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2}{cx^2+bx+a}\right.}{2\left(ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4+\left(b^4c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x),
1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^
2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [B] time = 1.20897, size = 729, normalized size = 6.39

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{2c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{2c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

Giac [A] time = 1.09743, size = 169, normalized size = 1.48

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{ab^2-2\,a^2c+\left(b^3-3\,abc\right)x}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3
)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (
b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

3.21 $\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal. Leaf size=67

$$\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0391678, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1585, 722, 618, 206}

$$\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^2}\,dx=\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx$$
$$=\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2a)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$
$$=\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4a)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}$$
$$=\frac{x(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.0937459, size = 81, normalized size = 1.21

$$\frac{a(b-2cx)+b^2x}{c\left(4ac-b^2\right)(a+x(b+cx))}+\frac{4a\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.006, size = 97, normalized size = 1.5

$$\frac{1}{cx^2+bx+a}\left(-\frac{\left(2\,ac-b^2\right)x}{c\left(4\,ac-b^2\right)}+\frac{ab}{c\left(4\,ac-b^2\right)}\right)+4\,\frac{a}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59667, size = 826, normalized size = 12.33

$$\left[-\frac{ab^3-4a^2bc+2\left(ac^2x^2+abcx+a^2c\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+\left(b^4-6ab^2c+8a^2c^2\right)x}{ab^4c-8a^2b^2c^2+16a^3c^3+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^2+\left(b^5c-8ab^3c^2+16a^2bc^3\right)x},-\frac{ab^3 \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 +
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x)]
```

Sympy [B] time = 0.755759, size = 280, normalized size = 4.18

$$-2a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2ab}{4ac}\right)+2a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (3
2*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) - (-a*b + x
*(2*a*c - b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*
(4*a*b*c**2 - b**3*c))
```

Giac [A] time = 1.11736, size = 119, normalized size = 1.78

$$-\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{b^2x-2\,acx+ab}{\left(b^2c-4\,ac^2\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

3.22 $\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^2} dx$

Optimal. Leaf size=66

$$\frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0378451, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1585, 638, 618, 206}

$$\frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^2}\,dx=\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx$$
$$=\frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$
$$=\frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}$$
$$=\frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.0677842, size = 69, normalized size = 1.05

$$\frac{2a+bx}{\left(b^2-4ac\right)(a+x(b+cx))}-\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.002, size = 70, normalized size = 1.1

$$\frac{-bx-2\,a}{\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)}-2\,\frac{b}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.53045, size = 740, normalized size = 11.21

$$\left[\frac{2\,ab^2-8\,a^2c-\left(bcx^2+b^2x+ab\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^3-4\,abc\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x},\frac{2\,ab^2-8\,a^2c-2\,(}{ab^4-8\,a^2b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 -
 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*x)]
```

Sympy [B] time = 0.705141, size = 252, normalized size = 3.82

$$b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)-b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2
)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)
**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**
2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) - (2*a + b*x)/(4*a**2*c - a*b*
*2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Giac [A] time = 1.14039, size = 103, normalized size = 1.56

$$\frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{bx+2\,a}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.23 $\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^2} \, dx$

Optimal. Leaf size=66

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0345985, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1585, 614, 618, 206}

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx$$
$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{(2c)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$
$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}$$
$$= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.0794278, size = 70, normalized size = 1.06

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{b+2cx}{a+x(b+cx)}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Maple [A] time = 0.003, size = 68, normalized size = 1.

$$\frac{2\,cx+b}{\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)} + 4\,\frac{c}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.55073, size = 745, normalized size = 11.29

$$\left[-\frac{b^3 - 4\,abc + 2\left(c^2x^2 + bcx + ac\right)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac - \sqrt{b^2 - 4\,ac}(2\,cx + b)}{cx^2 + bx + a}\right) + 2\left(b^2c - 4\,ac^2\right)x}{ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^2 + \left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)x}, -\frac{b^3 - 4\,abc - 4\left(c^2\right.}{ab^4 - 8\,a^2b^2c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 -
 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] time = 0.721547, size = 265, normalized size = 4.02

$$-2c\sqrt{-\frac{1}{\left(4ac - b^2\right)^3}}\log\left(x + \frac{-32a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} + 16ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} - 2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} + 2bc}{4c^2}\right) + 2c\sqrt{-\frac{1}{\left(4ac - b^2\right)^3}}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (
32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*
c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Giac [A] time = 1.10605, size = 103, normalized size = 1.56

$$-\frac{4\,c\arctan\left(\frac{2\,cx + b}{\sqrt{-b^2 + 4\,ac}}\right)}{\left(b^2 - 4\,ac\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,cx + b}{\left(cx^2 + bx + a\right)\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.24 $\int \frac{x^3}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rubi [A] time = 0.148149, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(ax^2+bx^3+cx^4\right)^2}\,dx &= \int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx \\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{-b^2+4ac-bcx}{x\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)} \\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-b^2+4ac}{ax}+\frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right)dx}{a\left(b^2-4ac\right)} \\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b\left(b^2-5ac\right)+c\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx}{a^2\left(b^2-4ac\right)} \\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2} - \frac{\left(b\left(b^2-6ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2\left(b^2-4ac\right)} \\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx+cx^2\right)}{2a^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x}\right.}{a^2\left(b^2-4ac\right)} \\
&= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx+cx^2\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.236263, size = 107, normalized size = 0.99

$$
\frac{\frac{2a\left(-2ac+b^2+bcx\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}-\log(a+x(b+cx))+2\log(x)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
- 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*L
og[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [B] time = 0.012, size = 237, normalized size = 2.2

$$\frac{\ln(x)}{a^2} - \frac{bcx}{a\left(cx^2+bx+a\right)\left(4ac-b^2\right)} + 2\frac{c}{\left(cx^2+bx+a\right)\left(4ac-b^2\right)} - \frac{b^2}{a\left(cx^2+bx+a\right)\left(4ac-b^2\right)} - 2\frac{c\ln\left(cx^2+bx+\right.}{a\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] ln(x)/a^2-1/a/(c*x^2+b*x+a)*b*c/(4*a*c-b^2)*x+2/(c*x^2+b*x+a)/(4*a*c-b^2)*c
-1/a/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2-2/a/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)+1/2/a^
2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b*c+1/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.09242, size = 1685, normalized size = 15.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2
 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^
3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
```

```
 - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2
+ (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

Sympy [B] time = 7.64518, size = 2236, normalized size = 20.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))*log(x + (1536*a**9*c**5*(-b
*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b*
*2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 2112*a**8*b**2*c**4*(-b*s
qrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 1136*a**7*b**4*c**3*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c
**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 768*a**7*c**5*(-b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - 1/(2*a**2)) - 300*a**6*b**6*c**2*(-b*sqrt(-(4*a*c - b*
*2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) - 1/(2*a**2))**2 + 624*a**6*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) - 1/(2*a**2)) + 39*a**5*b**8*c*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*
c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
- 1/(2*a**2))**2 - 184*a**5*b**4*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c -
 b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1
/(2*a**2)) - 768*a**5*c**5 - 2*a**4*b**10*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a
*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
 - 1/(2*a**2))**2 + 23*a**4*b**6*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c -
 b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1
/(2*a**2)) + 1488*a**4*b**2*c**4 - a**3*b**8*c*(-b*sqrt(-(4*a*c - b**2)**3)
*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)) - 1/(2*a**2)) - 952*a**3*b**4*c**3 + 277*a**2*b**6*c**2 - 38*a*b**8*c
 + 2*b**10)/(864*a**4*b*c**5 - 738*a**3*b**3*c**4 + 243*a**2*b**5*c**3 - 36
*a*b**7*c**2 + 2*b**9*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a
**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))*
log(x + (1536*a**9*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 -
 2112*a**8*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 11
36*a**7*b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a*
*3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 768*a
**7*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 -
 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 300*a**6*b**6*c**
2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 624*a**6*b**2*c**4*(b
*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b*
*2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) + 39*a**5*b**8*c*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 184*a**5*b**4*c**3*(b*sqrt(-(4*a*c -
 b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6)) - 1/(2*a**2)) - 768*a**5*c**5 - 2*a**4*b**10*(b*sqrt(-(4*a*
```

```
c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 23*a**4*b**6*c**2*(b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) - 1/(2*a**2)) + 1488*a**4*b**2*c**4 - a**3*b**8*c*(b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 952*a**3*b**4*c**3 + 277*a**2*b**6*c*
*2 - 38*a*b**8*c + 2*b**10)/(864*a**4*b*c**5 - 738*a**3*b**3*c**4 + 243*a**
2*b**5*c**3 - 36*a*b**7*c**2 + 2*b**9*c)) - (-2*a*c + b**2 + b*c*x)/(4*a**3
*c - a**2*b**2 + x**2*(4*a**2*c**2 - a*b**2*c) + x*(4*a**2*b*c - a*b**3)) +
 log(x)/a**2
```

Giac [A] time = 1.13067, size = 170, normalized size = 1.57

$$-\frac{\left(b^3 - 6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2b^2 - 4\,a^3c\right)\sqrt{-b^2 + 4\,ac}} - \frac{\log\left(cx^2 + bx + a\right)}{2\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{abcx + ab^2 - 2\,a^2c}{\left(cx^2 + bx + a\right)\left(b^2 - 4\,ac\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c
)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 + (a
*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

3.25 $\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^2} dx$

Optimal. Leaf size=148

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}+\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+\right.}$$

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 -
4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 +
(b*Log[a + b*x + c*x^2])/a^3
```

Rubi [A] time = 0.198026, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}+\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 -
4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 +
(b*Log[a + b*x + c*x^2])/a^3
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2} - \frac{2b\left(-b^2+4ac\right)}{a^2x} + \frac{2\left(-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4ac\right)x\right)}{a^2\left(a+bx+cx^2\right)}\right) d}{a\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} - \frac{2\int \frac{-b^4+5ab^2c-3a^2c^2-bc\left(b^2-4ac\right.}{a+bx+cx^2}}{a^3\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{a^3} + \frac{\left(b^4-6a\right.}{}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx+cx^2\right)}{a^3} - \frac{\left(2\right.}{}$$

$$= -\frac{2\left(b^2-3ac\right)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right.}{a^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.305082, size = 131, normalized size = 0.89

$$-\frac{\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}} + \frac{a\left(-3abc-2ac^2x+b^2cx+b^3\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)} - b\log(a+x(b+cx)) + \frac{a}{x} + 2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b
 + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3)
```

Maple [B] time = 0.013, size = 328, normalized size = 2.2

$$-\frac{1}{xa^2}-2\,\frac{b\ln(x)}{a^3}-2\,\frac{c^2x}{a\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}+\frac{cxb^2}{a^2\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-3\,\frac{bc}{a\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}+\frac{}{a^2\left(cx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/a^2/x-2*b*ln(x)/a^3-2/a/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x+1/a^2/(c*x^2+b*x
+a)*c/(4*a*c-b^2)*x*b^2-3/a/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c+1/a^2/(c*x^2+b*x+
a)*b^3/(4*a*c-b^2)+4/a^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*b-1/a^3/(4*a*c-b^2)*
ln(c*x^2+b*x+a)*b^3-12/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*c^2+12/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c-
2/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.61153, size = 2049, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3
*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
+ b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
```

```
^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c
- 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^
3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^
3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 1
6*a^6*c^2)*x)]
```

Sympy [B] time = 11.9409, size = 2672, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-1728*a*
*11*b*c**5*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b
**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 2
256*a**10*b**3*c**4*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b
**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
))**2 - 1172*a**9*b**5*c**3*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2
 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)))**2 - 288*a**9*c**6*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c*
*2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
*c - b**6))) + 303*a**8*b**7*c**2*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**
2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6)))**2 - 432*a**8*b**2*c**5*(b/a**3 - sqrt(-(4*a*c - b**2)**3)
*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6))) - 39*a**7*b**9*c*(b/a**3 - sqrt(-(4*a*c - b**2)**3)
*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)))**2 + 558*a**7*b**4*c**4*(b/a**3 - sqrt(-(4*a*c - b*
*2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6))) + 2*a**6*b**11*(b/a**3 - sqrt(-(4*a*c - b**2
)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6)))**2 - 212*a**6*b**6*c**3*(b/a**3 - sqrt(-(4*a*c
 - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6))) - 576*a**6*b*c**6 + 34*a**5*b**8*c**2*(
b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 6048*a**5*b**3*c
**5 - 2*a**4*b**10*c*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*
b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
))) - 7908*a**4*b**5*c**4 + 4264*a**3*b**7*c**3 - 1144*a**2*b**9*c**2 + 152
*a*b**11*c - 8*b**13)/(216*a**6*c**7 + 2808*a**5*b**2*c**6 - 5292*a**4*b**4
*c**5 + 3384*a**3*b**6*c**4 - 1008*a**2*b**8*c**3 + 144*a*b**10*c**2 - 8*b*
*12*c)) + (b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b*
*4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x +
 (-1728*a**11*b*c**5*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*
b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
)))**2 + 2256*a**10*b**3*c**4*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c*
*2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
```

```
*c - b**6)))**2 - 1172*a**9*b**5*c**3*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6
*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)))**2 - 288*a**9*c**6*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*
(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
 12*a*b**4*c - b**6))) + 303*a**8*b**7*c**2*(b/a**3 + sqrt(-(4*a*c - b**2)*
*3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)))**2 - 432*a**8*b**2*c**5*(b/a**3 + sqrt(-(4*a*c -
 b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6))) - 39*a**7*b**9*c*(b/a**3 + sqrt(-(4*a*c -
 b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 558*a**7*b**4*c**4*(b/a**3 + sqrt(-(
4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a**6*b**11*(b/a**3 + sqrt(-(4*
a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 212*a**6*b**6*c**3*(b/a**3 + sq
rt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**
3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 576*a**6*b*c**6 + 34*a**5*b
**8*c**2*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**
4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 6048*a
**5*b**3*c**5 - 2*a**4*b**10*c*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c
**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6))) - 7908*a**4*b**5*c**4 + 4264*a**3*b**7*c**3 - 1144*a**2*b**9*
c**2 + 152*a*b**11*c - 8*b**13)/(216*a**6*c**7 + 2808*a**5*b**2*c**6 - 5292
*a**4*b**4*c**5 + 3384*a**3*b**6*c**4 - 1008*a**2*b**8*c**3 + 144*a*b**10*c
**2 - 8*b**12*c)) - (4*a**2*c - a*b**2 + x**2*(6*a*c**2 - 2*b**2*c) + x*(7*
a*b*c - 2*b**3))/(x**3*(4*a**3*c**2 - a**2*b**2*c) + x**2*(4*a**3*b*c - a**
2*b**3) + x*(4*a**4*c - a**3*b**2)) - 2*b*log(x)/a**3
```

Giac [A] time = 1.08899, size = 231, normalized size = 1.56

$$\frac{2\left(b^{4}-6\,ab^{2}c+6\,a^{2}c^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{3}b^{2}-4\,a^{4}c\right)\sqrt{-b^{2}+4\,ac}}-\frac{2\,b^{2}cx^{2}-6\,ac^{2}x^{2}+2\,b^{3}x-7\,abcx+ab^{2}-4\,a^{2}c}{\left(a^{2}b^{2}-4\,a^{3}c\right)\left(cx^{3}+bx^{2}+ax\right)}+\frac{b\log\left(cx^{2}+bx+a\right)}{a^{3}}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x
 - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x))
 + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

3.26 $\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal. Leaf size=202

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}}-\frac{3b^2-8ac}{2a^2x^2\left(b^2-4ac\right)}-\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}+\frac{b\left(3b^2-11ac\right)}{a^3x\left(b^2-4ac\right)}+$$

```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b^2
 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)
) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 -
2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rubi [A] time = 0.250297, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}}-\frac{3b^2-8ac}{2a^2x^2\left(b^2-4ac\right)}-\frac{\left(3b^2-2ac\right)\log\left(a+bx+cx^2\right)}{2a^4}+\frac{b\left(3b^2-11ac\right)}{a^3x\left(b^2-4ac\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```


```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b^2
 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)
) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a
*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 -
2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{x^3\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \frac{-3b^2+8ac-3bcx}{x^3\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-3b^2+8ac}{ax^3} + \frac{3b^3-11abc}{a^2x^2} + \frac{\left(b^2-4ac\right)\left(-3b^2+2ac\right)}{a^3x} + \frac{b\left(3b^4-17ab^2c+19a^2c\ldots\right)}{a^3\left(a\ldots\right)}\right)}{a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log(x)}{a^4} - \ldots$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log(x)}{a^4} - \ldots$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{\left(3b^2-2ac\right)\log(x)}{a^4} - \ldots$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b\left(3b^2-11ac\right)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{b\left(3b^4-20ab^2c+30\ldots\right)}{a^4\left(b^2\ldots\right)}$$

Mathematica [A] time = 0.427668, size = 175, normalized size = 0.87

$$\frac{\frac{2a\left(2a^2c^2-4ab^2c-3abc^2x+b^3cx+b^4\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)}+\frac{2b\left(30a^2c^2-20ab^2c+3b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}-\frac{a^2}{x^2}+2\log(x)\left(3b^2-2ac\right)+\left(2ac-3b^2\right)\log(a+x(b+}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a
*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 3
0*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

Maple [B] time = 0.016, size = 418, normalized size = 2.1

$$-\frac{1}{2\,a^2x^2}-2\,\frac{\ln(x)\,c}{a^3}+3\,\frac{\ln(x)\,b^2}{a^4}+2\,\frac{b}{a^3x}+3\,\frac{c^2bx}{a^2\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-\frac{b^3cx}{a^3\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-2\,\frac{}{a\left(cx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/2/a^2/x^2-2/a^3*ln(x)*c+3/a^4*ln(x)*b^2+2/a^3*b/x+3/a^2/(c*x^2+b*x+a)*b*
c^2/(4*a*c-b^2)*x-1/a^3/(c*x^2+b*x+a)*b^3*c/(4*a*c-b^2)*x-2/a/(c*x^2+b*x+a)
/(4*a*c-b^2)*c^2+4/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c-1/a^3/(c*x^2+b*x+a)/
(4*a*c-b^2)*b^4+4/a^2/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)-7/a^3/(4*a*c-b^2)*c*l
n(c*x^2+b*x+a)*b^2+3/2/a^4/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^4+30/a^2/(4*a*c-b^
2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^2-20/a^3/(4*a*c-b^2)^(3/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c+3/a^4/(4*a*c-b^2)^(3/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.11668, size = 2606, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c
+ 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c
*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x +
((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a
*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*
a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^
4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3
*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*
c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a
*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)
*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*
a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2
)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a
^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3
*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)]
```

Sympy [B] time = 19.1743, size = 4083, normalized size = 20.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)
/(2*a**4))*log(x + (3072*a**14*c**6*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c
**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 9408*a**13*b**2*c**5*(-
b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(6
4*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(
2*a**4))**2 + 9040*a**12*b**4*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c*
*2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4116*a**11*b**6*c**3*(-b
*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2
*a**4))**2 + 3072*a**11*c**7*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 2
0*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 987*a**10*b**8*c**2*(-b*sqrt(-(4*
a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2
 - 7536*a**10*b**2*c**6*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b
**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 121*a**9*b**10*c*(-b*sqrt(-(4*a*c - b*
*2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a*
*2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 + 8152*
```

```
a**9*b**4*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3
*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (
2*a*c - 3*b**2)/(2*a**4)) + 6*a**8*b**12*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a
**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4343*a**8*b**6*c**
4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**
4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**
2)/(2*a**4)) - 6144*a**8*c**8 + 1198*a**7*b**8*c**3*(-b*sqrt(-(4*a*c - b**2
)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 50208*a**7
*b**2*c**7 - 165*a**6*b**10*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2
 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 137792*a**6*b**4*c**6 + 9*a**
5*b**12*c*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4
)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c
 - 3*b**2)/(2*a**4)) + 176474*a**5*b**6*c**5 - 119275*a**4*b**8*c**4 + 4544
8*a**3*b**10*c**3 - 9846*a**2*b**12*c**2 + 1134*a*b**14*c - 54*b**16)/(1728
0*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a**5*b**5*c**6 - 88605*a**4*b
**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**11*c**3 + 1080*a*b**13*c**2
- 54*b**15*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c +
3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
(2*a*c - 3*b**2)/(2*a**4))*log(x + (3072*a**14*c**6*(b*sqrt(-(4*a*c - b**2)
**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 9408*a**
13*b**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b*
*4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a
*c - 3*b**2)/(2*a**4))**2 + 9040*a**12*b**4*c**4*(b*sqrt(-(4*a*c - b**2)**3
)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4116*a**11*
b**6*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)
/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c
- 3*b**2)/(2*a**4))**2 + 3072*a**11*c**7*(b*sqrt(-(4*a*c - b**2)**3)*(30*a*
*2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 987*a**10*b**8*c**2*(b
*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2
*a**4))**2 - 7536*a**10*b**2*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2
 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 121*a**9*b**10*c*(b*sqrt(-(4*
a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2
 + 8152*a**9*b**4*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**
2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6)) + (2*a*c - 3*b**2)/(2*a**4)) + 6*a**8*b**12*(b*sqrt(-(4*a*c - b**2)**3)
*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4343*a**8*b*
*6*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(
2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c -
3*b**2)/(2*a**4)) - 6144*a**8*c**8 + 1198*a**7*b**8*c**3*(b*sqrt(-(4*a*c -
b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 50208*
a**7*b**2*c**7 - 165*a**6*b**10*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c
**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 137792*a**6*b**4*c**6 + 9*
a**5*b**12*c*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b*
*4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a
*c - 3*b**2)/(2*a**4)) + 176474*a**5*b**6*c**5 - 119275*a**4*b**8*c**4 + 45
448*a**3*b**10*c**3 - 9846*a**2*b**12*c**2 + 1134*a*b**14*c - 54*b**16)/(17
280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a**5*b**5*c**6 - 88605*a**4
```

```
*b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**11*c**3 + 1080*a*b**13*c**
2 - 54*b**15*c)) + (-4*a**3*c + a**2*b**2 + x**3*(22*a*b*c**2 - 6*b**3*c) +
 x**2*(-8*a**2*c**2 + 25*a*b**2*c - 6*b**4) + x*(12*a**2*b*c - 3*a*b**3))/(
x**4*(8*a**4*c**2 - 2*a**3*b**2*c) + x**3*(8*a**4*b*c - 2*a**3*b**3) + x**2
*(8*a**5*c - 2*a**4*b**2)) - (2*a*c - 3*b**2)*log(x + (-6144*a**8*c**8 + 50
208*a**7*b**2*c**7 - 3072*a**7*c**7*(2*a*c - 3*b**2) - 137792*a**6*b**4*c**
6 + 7536*a**6*b**2*c**6*(2*a*c - 3*b**2) + 3072*a**6*c**6*(2*a*c - 3*b**2)*
*2 + 176474*a**5*b**6*c**5 - 8152*a**5*b**4*c**5*(2*a*c - 3*b**2) - 9408*a*
*5*b**2*c**5*(2*a*c - 3*b**2)**2 - 119275*a**4*b**8*c**4 + 4343*a**4*b**6*c
**4*(2*a*c - 3*b**2) + 9040*a**4*b**4*c**4*(2*a*c - 3*b**2)**2 + 45448*a**3
*b**10*c**3 - 1198*a**3*b**8*c**3*(2*a*c - 3*b**2) - 4116*a**3*b**6*c**3*(2
*a*c - 3*b**2)**2 - 9846*a**2*b**12*c**2 + 165*a**2*b**10*c**2*(2*a*c - 3*b
**2) + 987*a**2*b**8*c**2*(2*a*c - 3*b**2)**2 + 1134*a*b**14*c - 9*a*b**12*
c*(2*a*c - 3*b**2) - 121*a*b**10*c*(2*a*c - 3*b**2)**2 - 54*b**16 + 6*b**12
*(2*a*c - 3*b**2)**2)/(17280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a*
*5*b**5*c**6 - 88605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**1
1*c**3 + 1080*a*b**13*c**2 - 54*b**15*c))/a**4
```

<u>Giac</u> [A] time = 1.12117, size = 309, normalized size = 1.53

$$-\frac{\left(3\,b^5-20\,ab^3c+30\,a^2bc^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^4b^2-4\,a^5c\right)\sqrt{-b^2+4\,ac}}-\frac{\left(3\,b^2-2\,ac\right)\log\left(cx^2+bx+a\right)}{2\,a^4}+\frac{\left(3\,b^2-2\,ac\right)\log\left(|x|\right)}{a^4}-\frac{a^3b^2-4\,a^4c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c -
2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2
 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

3.27 $\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal. Leaf size=252

$$-\frac{2\left(5a^2c^2-9ab^2c+2b^4\right)}{a^4x\left(b^2-4ac\right)}-\frac{2\left(30a^2b^2c^2-10a^3c^3-15ab^4c+2b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5\left(b^2-4ac\right)^{3/2}}+\frac{b\left(2b^2-7ac\right)}{a^3x^2\left(b^2-4ac\right)}-\frac{2\left(2b^2-5a\right.}{3a^2x^3\left(b^2-4\right.}$$

```
[Out] (-2*(2*b^2 - 5*a*c))/(3*a^2*(b^2 - 4*a*c)*x^3) + (b*(2*b^2 - 7*a*c))/(a^3*(
b^2 - 4*a*c)*x^2) - (2*(2*b^4 - 9*a*b^2*c + 5*a^2*c^2))/(a^4*(b^2 - 4*a*c)*
x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^3*(a + b*x + c*x^2)) - (2*(2*
b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)^(3/2)) - (2*b*(2*b^2 - 3*a*c)*Log[x])/a^5 +
 (b*(2*b^2 - 3*a*c)*Log[a + b*x + c*x^2])/a^5
```

Rubi [A] time = 0.323337, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1594, 740, 800, 634, 618, 206, 628}

$$-\frac{2\left(5a^2c^2-9ab^2c+2b^4\right)}{a^4x\left(b^2-4ac\right)}-\frac{2\left(30a^2b^2c^2-10a^3c^3-15ab^4c+2b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5\left(b^2-4ac\right)^{3/2}}+\frac{b\left(2b^2-7ac\right)}{a^3x^2\left(b^2-4ac\right)}-\frac{2\left(2b^2-5a\right.}{3a^2x^3\left(b^2-4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] (-2*(2*b^2 - 5*a*c))/(3*a^2*(b^2 - 4*a*c)*x^3) + (b*(2*b^2 - 7*a*c))/(a^3*(
b^2 - 4*a*c)*x^2) - (2*(2*b^4 - 9*a*b^2*c + 5*a^2*c^2))/(a^4*(b^2 - 4*a*c)*
x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^3*(a + b*x + c*x^2)) - (2*(2*
b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)^(3/2)) - (2*b*(2*b^2 - 3*a*c)*Log[x])/a^5 +
 (b*(2*b^2 - 3*a*c)*Log[a + b*x + c*x^2])/a^5
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{x^4\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+cx^2\right)} - \frac{\int \frac{-2\left(2b^2-5ac\right)-4bcx}{x^4\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{2\left(-2b^2+5ac\right)}{ax^4} - \frac{2\left(-2b^3+7abc\right)}{a^2x^3} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^3x^2} + \frac{2b\left(b^2-4ac\right)(2b}{a^4x}\right.}{a\left(b^2-\right.}$$

$$= -\frac{2\left(2b^2-5ac\right)}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\left(2b^2-7ac\right)}{a^3\left(b^2-4ac\right)x^2} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^4\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+\right.}$$

$$= -\frac{2\left(2b^2-5ac\right)}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\left(2b^2-7ac\right)}{a^3\left(b^2-4ac\right)x^2} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^4\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+\right.}$$

$$= -\frac{2\left(2b^2-5ac\right)}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\left(2b^2-7ac\right)}{a^3\left(b^2-4ac\right)x^2} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^4\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+\right.}$$

$$= -\frac{2\left(2b^2-5ac\right)}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\left(2b^2-7ac\right)}{a^3\left(b^2-4ac\right)x^2} - \frac{2\left(2b^4-9ab^2c+5a^2c^2\right)}{a^4\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+\right.}$$

Mathematica [A] time = 0.341258, size = 218, normalized size = 0.87

$$\frac{-\frac{3a\left(5a^2bc^2+2a^2c^3x-4ab^2c^2x-5ab^3c+b^4cx+b^5\right)}{\left(b^2-4ac\right)(a+x(b+cx))}-\frac{6\left(30a^2b^2c^2-10a^3c^3-15ab^4c+2b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{3a^2b}{x^2}-\frac{a^3}{x^3}+\frac{3a\left(2ac-3b^2\right)}{x}+6\log(x)\left(3ab\right.}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] (-(a^3/x^3) + (3*a^2*b)/x^2 + (3*a*(-3*b^2 + 2*a*c))/x - (3*a*(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + b^4*c*x - 4*a*b^2*c^2*x + 2*a^2*c^3*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (6*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*
ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 6*(-2*b^3 +
3*a*b*c)*Log[x] + 3*(2*b^3 - 3*a*b*c)*Log[a + x*(b + c*x)])/(3*a^5)
```

Maple [B] time = 0.016, size = 515, normalized size = 2.

$$-\frac{1}{3\,a^2x^3}+2\,\frac{c}{a^3x}-3\,\frac{b^2}{a^4x}+\frac{b}{x^2a^3}+6\,\frac{b\ln(x)\,c}{a^4}-4\,\frac{b^3\ln(x)}{a^5}+2\,\frac{c^3x}{a^2\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-4\,\frac{c^2xb^2}{a^3\left(cx^2+bx+a\right)\left(4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/3/a^2/x^3+2/a^3/x*c-3/a^4/x*b^2+1/a^3*b/x^2+6*b/a^4*ln(x)*c-4*b^3/a^5*ln
(x)+2/a^2/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x-4/a^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^
2)*x*b^2+1/a^4/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^4+5/a^2/(c*x^2+b*x+a)*b/(4*a
*c-b^2)*c^2-5/a^3/(c*x^2+b*x+a)*b^3/(4*a*c-b^2)*c+1/a^4/(c*x^2+b*x+a)*b^5/(
4*a*c-b^2)-12/a^3/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*b+11/a^4/(4*a*c-b^2)*c*ln
(c*x^2+b*x+a)*b^3-2/a^5/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^5+20/a^2/(4*a*c-b^2)^
(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3-60/a^3/(4*a*c-b^2)^(3/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2+30/a^4/(4*a*c-b^2)^(3/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*b^4*c-4/a^5/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.10804, size = 2989, normalized size = 11.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 +
 41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*
c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*
a^5*c^3)*x^2 - 3*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^
5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 1
5*a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^
5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b
^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 4
8*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a) + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^
2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a
^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*
x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a
^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3), -
1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 + 4
1*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*c^
2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*a^
5*c^3)*x^2 + 6*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^5
+ (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 15*
a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16
*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)
*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^
7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a)
 + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8
 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^
5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c
^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^
4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3)]
```

Sympy [B] time = 31.0241, size = 4774, normalized size = 18.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**
2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)))*log(x + (-4928*a**16*b*c**6*(-b*(3*a*c - 2*b**2)/a
**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**
4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
))**2 + 10032*a**15*b**3*c**5*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b*
*2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 7980*a**14*b**5
*c**4*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 -
30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)))**2 + 3249*a**13*b**7*c**3*(-b*(3*a*c - 2*b**
2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a
*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)))**2 + 800*a**13*c**8*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)
**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a*
*3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 723*a**12*b**9*c**2*(
```

```
-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2
*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)))**2 + 6704*a**12*b**2*c**7*(-b*(3*a*c - 2*b**2)/a**5
 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c
 - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))
+ 84*a**11*b**11*c*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10
*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3
- 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 15182*a**11*b**4*c**6*(-b*
(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b*
*2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6))) - 4*a**10*b**13*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a
*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a
**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 12844*a*
*10*b**6*c**5*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3
*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 5546*a**9*b**8*c**4*(-b*(3*a*c - 2
*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 +
15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6))) - 4800*a**9*b*c**9 + 1306*a**8*b**10*c**3*(-b*(3*a*c - 2*b**2)/a
**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**
4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
)) + 140384*a**8*b**3*c**8 - 160*a**7*b**12*c**2*(-b*(3*a*c - 2*b**2)/a**5
- sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c
- 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) -
 479788*a**7*b**5*c**7 + 8*a**6*b**14*c*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(
4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)
/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 709872*a
**6*b**7*c**6 - 575864*a**5*b**9*c**5 + 279640*a**4*b**11*c**4 - 83528*a**3
*b**13*c**3 + 15056*a**2*b**15*c**2 - 1504*a*b**17*c + 64*b**19)/(1000*a**9
*c**10 + 42840*a**8*b**2*c**9 - 232020*a**7*b**4*c**8 + 431760*a**6*b**6*c*
*7 - 406368*a**5*b**8*c**6 + 219600*a**4*b**10*c**5 - 71160*a**3*b**12*c**4
 + 13680*a**2*b**14*c**3 - 1440*a*b**16*c**2 + 64*b**18*c)) + (-b*(3*a*c -
2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 +
 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6)))*log(x + (-4928*a**16*b*c**6*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(
4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)
/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 10032
*a**15*b**3*c**5*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a
**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 -
48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 7980*a**14*b**5*c**4*(-b*(3*
a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*
c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6)))**2 + 3249*a**13*b**7*c**3*(-b*(3*a*c - 2*b**2)/a**5 + sqr
t(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b
**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 8
00*a**13*c**8*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3
*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 723*a**12*b**9*c**2*(-b*(3*a*c - 2
*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 +
15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)))**2 + 6704*a**12*b**2*c**7*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a
*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a
**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 84*a**11*b*
*11*c*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 -
30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)))**2 - 15182*a**11*b**4*c**6*(-b*(3*a*c - 2*b*
*2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*
a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6))) - 4*a**10*b**13*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3
```

```
)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*
c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 12844*a**10*b**6*c**5
*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a*
*2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**
2 + 12*a*b**4*c - b**6))) - 5546*a**9*b**8*c**4*(-b*(3*a*c - 2*b**2)/a**5 +
 sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c -
 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) -
4800*a**9*b*c**9 + 1306*a**8*b**10*c**3*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(
4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)
/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 140384*a
**8*b**3*c**8 - 160*a**7*b**12*c**2*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*
c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a*
*5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 479788*a**7*
b**5*c**7 + 8*a**6*b**14*c*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)
**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a*
*3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 709872*a**6*b**7*c**6
 - 575864*a**5*b**9*c**5 + 279640*a**4*b**11*c**4 - 83528*a**3*b**13*c**3 +
 15056*a**2*b**15*c**2 - 1504*a*b**17*c + 64*b**19)/(1000*a**9*c**10 + 4284
0*a**8*b**2*c**9 - 232020*a**7*b**4*c**8 + 431760*a**6*b**6*c**7 - 406368*a
**5*b**8*c**6 + 219600*a**4*b**10*c**5 - 71160*a**3*b**12*c**4 + 13680*a**2
*b**14*c**3 - 1440*a*b**16*c**2 + 64*b**18*c)) + (-4*a**4*c + a**3*b**2 + x
**4*(30*a**2*c**3 - 54*a*b**2*c**2 + 12*b**4*c) + x**3*(51*a**2*b*c**2 - 60
*a*b**3*c + 12*b**5) + x**2*(20*a**3*c**2 - 29*a**2*b**2*c + 6*a*b**4) + x*
(8*a**3*b*c - 2*a**2*b**3))/(x**5*(12*a**5*c**2 - 3*a**4*b**2*c) + x**4*(12
*a**5*b*c - 3*a**4*b**3) + x**3*(12*a**6*c - 3*a**5*b**2)) + 2*b*(3*a*c - 2
*b**2)*log(x + (-4800*a**9*b*c**9 + 140384*a**8*b**3*c**8 + 1600*a**8*b*c**
8*(3*a*c - 2*b**2) - 479788*a**7*b**5*c**7 + 13408*a**7*b**3*c**7*(3*a*c -
2*b**2) + 709872*a**6*b**7*c**6 - 30364*a**6*b**5*c**6*(3*a*c - 2*b**2) - 1
9712*a**6*b**3*c**6*(3*a*c - 2*b**2)**2 - 575864*a**5*b**9*c**5 + 25688*a**
5*b**7*c**5*(3*a*c - 2*b**2) + 40128*a**5*b**5*c**5*(3*a*c - 2*b**2)**2 + 2
79640*a**4*b**11*c**4 - 11092*a**4*b**9*c**4*(3*a*c - 2*b**2) - 31920*a**4*
b**7*c**4*(3*a*c - 2*b**2)**2 - 83528*a**3*b**13*c**3 + 2612*a**3*b**11*c**
3*(3*a*c - 2*b**2) + 12996*a**3*b**9*c**3*(3*a*c - 2*b**2)**2 + 15056*a**2*
b**15*c**2 - 320*a**2*b**13*c**2*(3*a*c - 2*b**2) - 2892*a**2*b**11*c**2*(3
*a*c - 2*b**2)**2 - 1504*a*b**17*c + 16*a*b**15*c*(3*a*c - 2*b**2) + 336*a*
b**13*c*(3*a*c - 2*b**2)**2 + 64*b**19 - 16*b**15*(3*a*c - 2*b**2)**2)/(100
0*a**9*c**10 + 42840*a**8*b**2*c**9 - 232020*a**7*b**4*c**8 + 431760*a**6*b
**6*c**7 - 406368*a**5*b**8*c**6 + 219600*a**4*b**10*c**5 - 71160*a**3*b**1
2*c**4 + 13680*a**2*b**14*c**3 - 1440*a*b**16*c**2 + 64*b**18*c))/a**5
```

Giac [A] time = 1.10674, size = 381, normalized size = 1.51

$$\frac{2\left(2\,b^6-15\,ab^4c+30\,a^2b^2c^2-10\,a^3c^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^5b^2-4\,a^6c\right)\sqrt{-b^2+4\,ac}}+\frac{\left(2\,b^3-3\,abc\right)\log\left(cx^2+bx+a\right)}{a^5}-\frac{2\left(2\,b^3-3\,abc\right)\log\left(|x|\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/((a^5*b^2 - 4*a^6*c)*sqrt(-b^2 + 4*a*c)) + (2*b^3 - 3*a*b*
c)*log(c*x^2 + b*x + a)/a^5 - 2*(2*b^3 - 3*a*b*c)*log(abs(x))/a^5 - 1/3*(a^
4*b^2 - 4*a^5*c + 6*(2*a*b^4*c - 9*a^2*b^2*c^2 + 5*a^3*c^3)*x^4 + 3*(4*a*b^
5 - 20*a^2*b^3*c + 17*a^3*b*c^2)*x^3 + (6*a^2*b^4 - 29*a^3*b^2*c + 20*a^4*c
^2)*x^2 - 2*(a^3*b^3 - 4*a^4*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^5*x
^3)
```

3.28 $\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal. Leaf size=318

$$-\frac{12a^2c^2-22ab^2c+5b^4}{2a^4x^2\left(b^2-4ac\right)}-\frac{\left(3a^2c^2-12ab^2c+5b^4\right)\log\left(a+bx+cx^2\right)}{2a^6}+\frac{b\left(29a^2c^2-27ab^2c+5b^4\right)}{a^5x\left(b^2-4ac\right)}+\frac{\log(x)\left(3a^2c^2-1\right.}{a^6}$$

```
[Out] -(5*b^2 - 12*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(5*b^2 - 17*a*c))/(3*a^3*(
b^2 - 4*a*c)*x^3) - (5*b^4 - 22*a*b^2*c + 12*a^2*c^2)/(2*a^4*(b^2 - 4*a*c)*
x^2) + (b*(5*b^4 - 27*a*b^2*c + 29*a^2*c^2))/(a^5*(b^2 - 4*a*c)*x) + (b^2 -
 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^4*(a + b*x + c*x^2)) + (b*(5*b^6 - 42*a*
b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(a^6*(b^2 - 4*a*c)^(3/2)) + ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[x])/a^
6 - ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + b*x + c*x^2])/(2*a^6)
```

Rubi [A] time = 0.391815, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$-\frac{12a^2c^2-22ab^2c+5b^4}{2a^4x^2\left(b^2-4ac\right)}-\frac{\left(3a^2c^2-12ab^2c+5b^4\right)\log\left(a+bx+cx^2\right)}{2a^6}+\frac{b\left(29a^2c^2-27ab^2c+5b^4\right)}{a^5x\left(b^2-4ac\right)}+\frac{\log(x)\left(3a^2c^2-1\right.}{a^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```


```
[Out] -(5*b^2 - 12*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(5*b^2 - 17*a*c))/(3*a^3*(
b^2 - 4*a*c)*x^3) - (5*b^4 - 22*a*b^2*c + 12*a^2*c^2)/(2*a^4*(b^2 - 4*a*c)*
x^2) + (b*(5*b^4 - 27*a*b^2*c + 29*a^2*c^2))/(a^5*(b^2 - 4*a*c)*x) + (b^2 -
 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^4*(a + b*x + c*x^2)) + (b*(5*b^6 - 42*a*
b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(a^6*(b^2 - 4*a*c)^(3/2)) + ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[x])/a^
6 - ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + b*x + c*x^2])/(2*a^6)
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{x^5\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} - \frac{\int \frac{-5b^2+12ac-5bcx}{x^5\left(a+bx+cx^2\right)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-5b^2+12ac}{ax^5} + \frac{5b^3-17abc}{a^2x^4} + \frac{-5b^4+22ab^2c-12a^2c^2}{a^3x^3} + \frac{5b^5-27ab^3c+29a}{a^4x^2}\right.}{}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(5b^2-17ac\right)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b\left(5b^4-27ab^2c+29a^2c\right.}{a^5\left(b^2-4ac\right)x}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(5b^2-17ac\right)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b\left(5b^4-27ab^2c+29a^2c\right.}{a^5\left(b^2-4ac\right)x}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(5b^2-17ac\right)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b\left(5b^4-27ab^2c+29a^2c\right.}{a^5\left(b^2-4ac\right)x}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b\left(5b^2-17ac\right)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b\left(5b^4-27ab^2c+29a^2c\right.}{a^5\left(b^2-4ac\right)x}$$

Mathematica [A] time = 0.413161, size = 272, normalized size = 0.86

$$\frac{-\frac{12a\left(-9a^2b^2c^2-5a^2bc^3x+2a^3c^3+5ab^3c^2x+6ab^4c-b^5cx-b^6\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+12\log(x)\left(3a^2c^2-12ab^2c+5b^4\right)-6\left(3a^2c^2-12ab^2c+5b^4\right)\log(a+x}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

```
[Out] ((-3*a^4)/x^4 + (8*a^3*b)/x^3 + (6*a^2*(-3*b^2 + 2*a*c))/x^2 - (24*a*b*(-2*
b^2 + 3*a*c))/x - (12*a*(-b^6 + 6*a*b^4*c - 9*a^2*b^2*c^2 + 2*a^3*c^3 - b^5
*c*x + 5*a*b^3*c^2*x - 5*a^2*b*c^3*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) +
(12*b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTan[(b + 2*c*x
)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 12*(5*b^4 - 12*a*b^2*c + 3*a^
2*c^2)*Log[x] - 6*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + x*(b + c*x)])/(1
2*a^6)
```

Maple [B] time = 0.018, size = 619, normalized size = 2.

$$-\frac{1}{4a^2x^4}+\frac{c}{x^2a^3}-\frac{3b^2}{2a^4x^2}+3\frac{\ln(x)c^2}{a^4}-12\frac{\ln(x)b^2c}{a^5}+5\frac{\ln(x)b^4}{a^6}+\frac{2b}{3a^3x^3}-6\frac{bc}{a^4x}+4\frac{b^3}{a^5x}-5\frac{bc^3x}{a^3\left(cx^2+bx+a\right)(4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/4/a^2/x^4+1/a^3/x^2*c-3/2/a^4/x^2*b^2+3/a^4*ln(x)*c^2-12/a^5*ln(x)*b^2*c
+5/a^6*ln(x)*b^4+2/3/a^3*b/x^3-6*b/a^4/x*c+4*b^3/a^5/x-5/a^3/(c*x^2+b*x+a)*
b*c^3/(4*a*c-b^2)*x+5/a^4/(c*x^2+b*x+a)*b^3*c^2/(4*a*c-b^2)*x-1/a^5/(c*x^2+
b*x+a)*b^5*c/(4*a*c-b^2)*x+2/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*c^3-9/a^3/(c*x^2
+b*x+a)/(4*a*c-b^2)*b^2*c^2+6/a^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4*c-1/a^5/(c*
x^2+b*x+a)/(4*a*c-b^2)*b^6-6/a^3/(4*a*c-b^2)*c^3*ln(c*x^2+b*x+a)+51/2/a^4/(
4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*b^2-16/a^5/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*b^4
+5/2/a^6/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^6-70/a^3/(4*a*c-b^2)^(3/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3+105/a^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*b^3*c^2-42/a^5/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*b^5*c+5/a^6/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*b^7
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.72475, size = 3563, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*
c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 3
16*a^3*b^4*c^2 - 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^
3*b^5*c + 448*a^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c
 + 232*a^5*b^2*c^2 - 144*a^6*c^3)*x^2 - 6*((5*b^7*c - 42*a*b^5*c^2 + 105*a^
2*b^3*c^3 - 70*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*
a^3*b^2*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3
)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
- 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 5*(a^4*b^5 - 8*a^5*b^3*c + 16*a^
6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4
 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^
3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4
*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) - 12*((5*b^8*c - 52*a*b^6*
c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b
^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 5
2*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(x))/
((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a^6*b^5 - 8*a^7*b^3*c + 16
*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*x^4), -1/12*(3*a^5*b
^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*c^2 + 137*a^3*b
^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 316*a^3*b^4*c^2
- 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^3*b^5*c + 448*a
^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c + 232*a^5*b^2*
c^2 - 144*a^6*c^3)*x^2 - 12*((5*b^7*c - 42*a*b^5*c^2 + 105*a^2*b^3*c^3 - 70
*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*a^3*b^2*c^3)*x
^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3)*x^4)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 5*(a^4*
b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*
b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2
*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c +
179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) -
 12*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c
^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*
b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 +
48*a^5*c^4)*x^4)*log(x))/((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a
^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)*x^4)]
```

Sympy [B] time = 47.4723, size = 6181, normalized size = 19.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4
*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))*log(x + (4608*a**19*c**
```

```
7*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b*
*4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b*
*6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 26432*a**18*b**2
*c**6*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*
a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 38640*a**17*
b**4*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 +
 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 26124*a*
*16*b**6*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c*
*2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 9603
*a**15*b**8*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2
*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 6
912*a**15*c**9*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c
**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 1989*a
**14*b**10*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*
c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 37
616*a**14*b**2*c**8*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b
**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) + 2
19*a**13*b**12*c*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2
*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 9
6472*a**13*b**4*c**7*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*
b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) -
10*a**12*b**14*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c
**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 112
063*a**12*b**6*c**6*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b
**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 6
9023*a**11*b**8*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*
b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) -
20736*a**11*c**11 + 24355*a**10*b**10*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(70
*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c*
*3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c
 + 5*b**4)/(2*a**6)) + 373872*a**10*b**2*c**10 - 4964*a**9*b**12*c**3*(-b*sq
rt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5
*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (
3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 2277288*a**9*b**4*c**9 + 54
5*a**8*b**14*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**
2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) + 648
7391*a**8*b**6*c**8 - 25*a**7*b**16*c*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3
*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 -
48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b
**4)/(2*a**6)) - 9943570*a**7*b**8*c**7 + 9090837*a**6*b**10*c**6 - 5264714
*a**5*b**12*c**5 + 1984426*a**4*b**14*c**4 - 486146*a**3*b**16*c**3 + 74720
*a**2*b**18*c**2 - 6550*a*b**20*c + 250*b**22)/(90720*a**10*b*c**11 - 84413
0*a**9*b**3*c**10 + 3174507*a**8*b**5*c**9 - 5885010*a**7*b**7*c**8 + 61682
25*a**6*b**9*c**7 - 3960180*a**5*b**11*c**6 + 1618470*a**4*b**13*c**5 - 423
276*a**3*b**15*c**4 + 68670*a**2*b**17*c**3 - 6300*a*b**19*c**2 + 250*b**21
*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*
```

```
a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))*log(x + (4608*a**
19*c**7*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42
*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 26432*a**18
*b**2*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 +
 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 38640*a*
*17*b**4*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**
2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 26124
*a**16*b**6*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*
c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 96
03*a**15*b**8*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**
2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 -
6912*a**15*c**9*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c
**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 1989*a
**14*b**10*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c
**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 376
16*a**14*b**2*c**8*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**
2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) + 219
*a**13*b**12*c*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c*
*2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 9647
2*a**13*b**4*c**7*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2
*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 10*a
**12*b**14*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 +
 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 112063*a
**12*b**6*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c*
*2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 69023*a
**11*b**8*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c*
*2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 20736*a
**11*c**11 + 24355*a**10*b**10*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c*
*3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4
)/(2*a**6)) + 373872*a**10*b**2*c**10 - 4964*a**9*b**12*c**3*(b*sqrt(-(4*a*
c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2
*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c*
*2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 2277288*a**9*b**4*c**9 + 545*a**8*b*
*14*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 4
2*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) + 6487391*a**8*
b**6*c**8 - 25*a**7*b**16*c*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105
*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**
6)) - 9943570*a**7*b**8*c**7 + 9090837*a**6*b**10*c**6 - 5264714*a**5*b**12
*c**5 + 1984426*a**4*b**14*c**4 - 486146*a**3*b**16*c**3 + 74720*a**2*b**18
*c**2 - 6550*a*b**20*c + 250*b**22)/(90720*a**10*b*c**11 - 844130*a**9*b**3
*c**10 + 3174507*a**8*b**5*c**9 - 5885010*a**7*b**7*c**8 + 6168225*a**6*b**
9*c**7 - 3960180*a**5*b**11*c**6 + 1618470*a**4*b**13*c**5 - 423276*a**3*b*
```

```
*15*c**4 + 68670*a**2*b**17*c**3 - 6300*a*b**19*c**2 + 250*b**21*c)) - (12*
a**5*c - 3*a**4*b**2 + x**5*(348*a**2*b*c**3 - 324*a*b**3*c**2 + 60*b**5*c)
 + x**4*(-72*a**3*c**3 + 480*a**2*b**2*c**2 - 354*a*b**4*c + 60*b**6) + x**
3*(208*a**3*b*c**2 - 172*a**2*b**3*c + 30*a*b**5) + x**2*(-36*a**4*c**2 + 4
9*a**3*b**2*c - 10*a**2*b**4) + x*(-20*a**4*b*c + 5*a**3*b**3))/(x**6*(48*a
**6*c**2 - 12*a**5*b**2*c) + x**5*(48*a**6*b*c - 12*a**5*b**3) + x**4*(48*a
**7*c - 12*a**6*b**2)) + (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)*log(x + (-207
36*a**11*c**11 + 373872*a**10*b**2*c**10 - 2277288*a**9*b**4*c**9 - 6912*a*
*9*c**9*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) + 6487391*a**8*b**6*c**8 + 376
16*a**8*b**2*c**8*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) - 9943570*a**7*b**8*
c**7 - 96472*a**7*b**4*c**7*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) + 4608*a**
7*c**7*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 + 9090837*a**6*b**10*c**6 +
112063*a**6*b**6*c**6*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) - 26432*a**6*b**
2*c**6*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 - 5264714*a**5*b**12*c**5 -
69023*a**5*b**8*c**5*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) + 38640*a**5*b**4
*c**5*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 + 1984426*a**4*b**14*c**4 + 2
4355*a**4*b**10*c**4*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) - 26124*a**4*b**6
*c**4*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 - 486146*a**3*b**16*c**3 - 49
64*a**3*b**12*c**3*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) + 9603*a**3*b**8*c*
*3*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 + 74720*a**2*b**18*c**2 + 545*a*
*2*b**14*c**2*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) - 1989*a**2*b**10*c**2*(
3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 - 6550*a*b**20*c - 25*a*b**16*c*(3*a
**2*c**2 - 12*a*b**2*c + 5*b**4) + 219*a*b**12*c*(3*a**2*c**2 - 12*a*b**2*c
 + 5*b**4)**2 + 250*b**22 - 10*b**14*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**
2)/(90720*a**10*b*c**11 - 844130*a**9*b**3*c**10 + 3174507*a**8*b**5*c**9 -
 5885010*a**7*b**7*c**8 + 6168225*a**6*b**9*c**7 - 3960180*a**5*b**11*c**6
+ 1618470*a**4*b**13*c**5 - 423276*a**3*b**15*c**4 + 68670*a**2*b**17*c**3
- 6300*a*b**19*c**2 + 250*b**21*c))/a**6
```

Giac [A] time = 1.12268, size = 468, normalized size = 1.47

$$-\frac{\left(5\,b^7-42\,ab^5c+105\,a^2b^3c^2-70\,a^3bc^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^6b^2-4\,a^7c\right)\sqrt{-b^2+4\,ac}}-\frac{\left(5\,b^4-12\,ab^2c+3\,a^2c^2\right)\log\left(cx^2+bx+a\right)}{2\,a^6}+\frac{\left(5\,b^4-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(5*b^7 - 42*a*b^5*c + 105*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*c*x + b)/s
qrt(-b^2 + 4*a*c))/((a^6*b^2 - 4*a^7*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(5*b^4 -
12*a*b^2*c + 3*a^2*c^2)*log(c*x^2 + b*x + a)/a^6 + (5*b^4 - 12*a*b^2*c + 3*
a^2*c^2)*log(abs(x))/a^6 - 1/12*(3*a^5*b^2 - 12*a^6*c - 12*(5*a*b^5*c - 27*
a^2*b^3*c^2 + 29*a^3*b*c^3)*x^5 - 6*(10*a*b^6 - 59*a^2*b^4*c + 80*a^3*b^2*c
^2 - 12*a^4*c^3)*x^4 - 2*(15*a^2*b^5 - 86*a^3*b^3*c + 104*a^4*b*c^2)*x^3 +
(10*a^3*b^4 - 49*a^4*b^2*c + 36*a^5*c^2)*x^2 - 5*(a^4*b^3 - 4*a^5*b*c)*x)/(
(c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^6*x^4)
```

3.29 $\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=257

$$-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}-\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx}}{960c^3}$$

```
[Out] (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^3) - ((105*b^4 -
460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(1920*c^4*x) - ((7*
b^2 - 16*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(240*c^2) + (x^2*(b + 8*c*x)*S
qrt[a*x^2 + b*x^3 + c*x^4])/(40*c) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sq
rt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])
/(256*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.588441, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1919, 1949, 12, 1914, 621, 206}

$$-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}-\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx}}{960c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^3) - ((105*b^4 -
460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(1920*c^4*x) - ((7*
b^2 - 16*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(240*c^2) + (x^2*(b + 8*c*x)*S
qrt[a*x^2 + b*x^3 + c*x^4])/(40*c) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sq
rt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])
/(256*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
```

```
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx = \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} + \frac{\int \frac{x^3\left(-3ab-\frac{1}{2}\left(7b^2-16ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{40c}$$

$$= -\frac{\left(7b^2-16ac\right)x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} - \frac{\int \frac{x^2\left(-a\left(7b^2-16ac\right)-\frac{1}{4}b(3\ldots}{\sqrt{ax^2+bx^3+\ldots}}}{120c^2}$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(7b^2-16ac\right)x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8cx)\sqrt{ax^\ldots}}{40c}$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} - \ldots$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} - \ldots$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} - \ldots$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} - \ldots$$

$$= \frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x} - \ldots$$

Mathematica [A] time = 0.241455, size = 180, normalized size = 0.7

$$\frac{2\sqrt{c}x(a+x(b+cx))\left(128c^2\left(-2a^2+acx^2+3c^2x^4\right)+4b^2c\left(115a-14cx^2\right)+8bc^2x\left(6cx^2-29a\right)+70b^3cx-105b^4\right)+15x\left(\right.}{3840c^{9/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-105*b^4 + 70*b^3*c*x + 4*b^2*c*(115*a - 14
*c*x^2) + 8*b*c^2*x*(-29*a + 6*c*x^2) + 128*c^2*(-2*a^2 + a*c*x^2 + 3*c^2*x
^4)) + 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*x*Sqrt[a + x*(b + c*x)]*Log[b
 + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(9/2)*Sqrt[x^2*(a + x*
(b + c*x))])
```

Maple [A] time = 0.007, size = 310, normalized size = 1.2

$$\frac{1}{3840\,x}\sqrt{cx^4+bx^3+ax^2}\left(768\,x^2\left(cx^2+bx+a\right)^{3/2}c^{9/2}-672\,c^{7/2}\left(cx^2+bx+a\right)^{3/2}xb-512\,c^{7/2}\left(cx^2+bx+a\right)^{3/2}a+560\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/3840*(c*x^4+b*x^3+a*x^2)^(1/2)*(768*x^2*(c*x^2+b*x+a)^(3/2)*c^(9/2)-672*c
^(7/2)*(c*x^2+b*x+a)^(3/2)*x*b-512*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a+560*c^(5/2
)*(c*x^2+b*x+a)^(3/2)*b^2+720*c^(7/2)*(c*x^2+b*x+a)^(1/2)*x*a*b-420*c^(5/2)
*(c*x^2+b*x+a)^(1/2)*x*b^3+360*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b^2-210*c^(3/2
)*(c*x^2+b*x+a)^(1/2)*b^4+720*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b
)/c^(1/2))*a^2*b*c^3-600*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(
1/2))*a*b^3*c^2+105*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))
*b^5*c)/x/(c*x^2+b*x+a)^(1/2)/c^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+bx^3+ax^2}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2, x)
```

Fricas [A] time = 1.73634, size = 898, normalized size = 3.49

$$\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+4\left(384\,c^5x^4+48\,bc^4x^3-105\,b^4c\right.}{7680\,c^5x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 +
 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a
*c)*x)/x) + 4*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460*a*b^2*c^2 - 256
*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/3840*(15*(7*b^5 - 40*a*b^3*c + 48*a
^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqr
t(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^
4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^
3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{x^2 \left(a + bx + cx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [A] time = 1.18191, size = 382, normalized size = 1.49

$$\frac{1}{1920}\sqrt{cx^2 + bx + a}\left(2\left(4\left(6\left(8\,x\mathrm{sgn}\left(x\right) + \frac{b\mathrm{sgn}\left(x\right)}{c}\right)x - \frac{7\,b^2c^2\mathrm{sgn}\left(x\right) - 16\,ac^3\mathrm{sgn}\left(x\right)}{c^4}\right)x + \frac{35\,b^3c\mathrm{sgn}\left(x\right) - 116\,abc^2\mathrm{sg}}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*x*sgn(x) + b*sgn(x)/c)*x - (7*b^2*
c^2*sgn(x) - 16*a*c^3*sgn(x))/c^4)*x + (35*b^3*c*sgn(x) - 116*a*b*c^2*sgn(x
))/c^4)*x - (105*b^4*sgn(x) - 460*a*b^2*c*sgn(x) + 256*a^2*c^2*sgn(x))/c^4)
 - 1/256*(7*b^5*sgn(x) - 40*a*b^3*c*sgn(x) + 48*a^2*b*c^2*sgn(x))*log(abs(-
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2) + 1/3840*(105*b
^5*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*log(abs(-b + 2*sqrt(a)*sq
rt(c))) + 720*a^2*b*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b^4*
sqrt(c) - 920*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5/2))*sgn(x)/c^(9/2)
```

3.30 $\int x\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=205

$$\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2}-\frac{x(b^2-4ac)(5b^2-4ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{128c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*c^2) + (b*(15*b^2 - 52*
a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*c^3*x) + (x*(b + 6*c*x)*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(24*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c
*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*
Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.370274, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1919, 1949, 12, 1914, 621, 206}

$$\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2}-\frac{x(b^2-4ac)(5b^2-4ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{128c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```



```
[Out] -((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*c^2) + (b*(15*b^2 - 52*
a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*c^3*x) + (x*(b + 6*c*x)*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(24*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c
*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*
Sqrt[a*x^2 + b*x^3 + c*x^4])
```


Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
```

```
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx = \frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c} + \frac{\int \frac{x^2\left(-2ab-\frac{1}{2}\left(5b^2-12ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{24c}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c} - \frac{\int \frac{x\left(-\frac{1}{2}a\left(5b^2-12ac\right)-\frac{1}{4}b\left(15b^2\right.\right.}{\sqrt{ax^2+bx^3+cx^4}}}{48c^2}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{ax^2+}}{24c}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{ax^2+}}{24c}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{ax^2+}}{24c}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{ax^2+}}{24c}$$

$$= -\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{ax^2+}}{24c}$$

Mathematica [A] time = 0.192789, size = 150, normalized size = 0.73

$$\frac{2\sqrt{c}x(a+x(b+cx))\left(b\left(8c^2x^2-52ac\right)+24c^2x\left(a+2cx^2\right)-10b^2cx+15b^3\right)-3x\left(16a^2c^2-24ab^2c+5b^4\right)\sqrt{a+x(b+}}{384c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(15*b^3 - 10*b^2*c*x + 24*c^2*x*(a + 2*c*x^2
) + b*(-52*a*c + 8*c^2*x^2)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x*Sqrt[a
 + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(7
/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.008, size = 265, normalized size = 1.3

$$\frac{1}{384\,x}\sqrt{cx^4+bx^3+ax^2}\left(96\,x\left(cx^2+bx+a\right)^{3/2}c^{7/2}-80\,c^{5/2}\left(cx^2+bx+a\right)^{3/2}b-48\,c^{7/2}\sqrt{cx^2+bx+a}xa+60\,c^{5/2}\sqrt{cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/384*(c*x^4+b*x^3+a*x^2)^(1/2)*(96*x*(c*x^2+b*x+a)^(3/2)*c^(7/2)-80*c^(5/2
)*(c*x^2+b*x+a)^(3/2)*b-48*c^(7/2)*(c*x^2+b*x+a)^(1/2)*x*a+60*c^(5/2)*(c*x^
2+b*x+a)^(1/2)*x*b^2-24*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b+30*c^(3/2)*(c*x^2+b
*x+a)^(1/2)*b^3-48*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*
a^2*c^3+72*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^2*c^
2-15*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^4*c)/x/(c*x^
2+b*x+a)^(1/2)/c^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^3+ax^2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x, x)
```

Fricas [A] time = 1.64592, size = 737, normalized size = 3.6

$$\left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+4\left(48\,c^4x^3+8\,bc^3x^2+15\,b^3c-52\,a\right.}{768\,c^4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b
*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*
x)/x) + 4*(48*c^4*x^3 + 8*b*c^3*x^2 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2
```

```
- 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x), 1/384*(3*(5*b^4 - 24*a
*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(48*c^4*x^3 + 8*b*c^3*x^2
 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 +
 a*x^2))/(c^4*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [A] time = 1.21113, size = 311, normalized size = 1.52

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,x\mathrm{sgn}\left(x\right)+\frac{b\mathrm{sgn}\left(x\right)}{c}\right)x-\frac{5\,b^2c\mathrm{sgn}\left(x\right)-12\,ac^2\mathrm{sgn}\left(x\right)}{c^3}\right)x+\frac{15\,b^3\mathrm{sgn}\left(x\right)-52\,abc\mathrm{sgn}\left(x\right)}{c^3}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*x*sgn(x) + b*sgn(x)/c)*x - (5*b^2*c*sg
n(x) - 12*a*c^2*sgn(x))/c^3)*x + (15*b^3*sgn(x) - 52*a*b*c*sgn(x))/c^3) + 1
/128*(5*b^4*sgn(x) - 24*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(-2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2) - 1/384*(15*b^4*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) +
 48*a^2*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 104
*a^(3/2)*b*c^(3/2))*sgn(x)/c^(7/2)
```

3.31 $\int \sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=163

$$\frac{b\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx}$$

```
[Out] -(b*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c^2*x) + ((a + b*x + c*x^2)
*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c*x) + (b*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3
 + c*x^4]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/
2)*x*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.0576031, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1903, 640, 612, 621, 206}

$$\frac{b\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c^2*x) + ((a + b*x + c*x^2)
*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c*x) + (b*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3
 + c*x^4]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/
2)*x*Sqrt[a + b*x + c*x^2])
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}\int x\sqrt{a+bx+cx^2}\,dx}{x\sqrt{a+bx+cx^2}}$$

$$= \frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx} - \frac{\left(b\sqrt{ax^2+bx^3+cx^4}\right)\int \sqrt{a+bx+cx^2}\,dx}{2cx\sqrt{a+bx+cx^2}}$$

$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{\left(b\left(b^2-4ac\right)\sqrt{ax^2+b\right.}{16c^2x\sqrt{a}$$

$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{\left(b\left(b^2-4ac\right)\sqrt{ax^2+b\right.}{}$$

$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{b\left(b^2-4ac\right)\sqrt{ax^2+bx}}{16c^{5/2}x}$$

Mathematica [A] time = 0.219505, size = 119, normalized size = 0.73

$$\frac{2\sqrt{c}x(a+x(b+cx))\left(8c\left(a+cx^2\right)-3b^2+2bcx\right)+3bx\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{48c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-3*b^2 + 2*b*c*x + 8*c*(a + c*x^2)) + 3*b*(
b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])])/(48*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.006, size = 167, normalized size = 1.

$$\frac{1}{48\,x}\sqrt{cx^4+bx^3+ax^2}\left(16\left(cx^2+bx+a\right)^{3/2}c^{5/2}-12\,c^{5/2}\sqrt{cx^2+bx+a}xb-6\,c^{3/2}\sqrt{cx^2+bx+a}b^2-12\,\ln\left(1/2\,\frac{2\sqrt{cx^2}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(1/2)*(16*(c*x^2+b*x+a)^(3/2)*c^(5/2)-12*c^(5/2)*(
c*x^2+b*x+a)^(1/2)*x*b-6*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^2-12*ln(1/2*(2*(c*x^
2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b*c^2+3*ln(1/2*(2*(c*x^2+b*x+a)^
(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^3*c)/x/(c*x^2+b*x+a)^(1/2)/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^3 + ax^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Fricas [A] time = 1.61506, size = 591, normalized size = 3.63

$$\left[-\frac{3\left(b^3 - 4\,abc\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3 + 8\,bcx^2 - 4\,\sqrt{cx^4 + bx^3 + ax^2}(2\,cx + b)\sqrt{c} + \left(b^2 + 4\,ac\right)x}{x}\right) - 4\left(8\,c^3x^2 + 2\,bc^2x - 3\,b^2c + 8\,ac^2\right)\sqrt{cx^4 + bx^3 + \ldots}}{96\,c^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*
x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(8*c^3*x
^2 + 2*b*c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -
1/48*(3*(b^3 - 4*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(8*c^3*x^2 + 2*b*c^2*x
 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^2 + bx^3 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

Giac [A] time = 1.14418, size = 224, normalized size = 1.37

$$\frac{1}{24}\sqrt{cx^2 + bx + a}\left(2\left(4\,x\mathrm{sgn}\left(x\right) + \frac{b\mathrm{sgn}\left(x\right)}{c}\right)x - \frac{3\,b^2\mathrm{sgn}\left(x\right) - 8\,ac\mathrm{sgn}\left(x\right)}{c^2}\right) - \frac{\left(b^3\mathrm{sgn}\left(x\right) - 4\,abc\mathrm{sgn}\left(x\right)\right)\log\left(\left|-2\left(\sqrt{c}x - \ldots\right.\right.\right.}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*x*sgn(x) + b*sgn(x)/c)*x - (3*b^2*sgn(x) -
 8*a*c*sgn(x))/c^2) - 1/16*(b^3*sgn(x) - 4*a*b*c*sgn(x))*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) + 1/48*(3*b^3*log(abs(-b
 + 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt
(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/c^(5/2)
```

3.32 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x} dx$

Optimal. Leaf size=119

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[a
 + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*
c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.0777735, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1918, 1914, 621, 206}

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

```
[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[a
 + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*
c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx &= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left(b^2-4ac\right)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c} \\
&= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left(\left(b^2-4ac\right)x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left(\left(b^2-4ac\right)x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4c\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left(b^2-4ac\right)x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.14577, size = 100, normalized size = 0.84

$$\frac{x\left(2\sqrt{c}(b+2cx)(a+x(b+cx))-\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\right)}{8c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

```
[Out] (x*(2*Sqrt[c]*(b + 2*c*x)*(a + x*(b + c*x)) - (b^2 - 4*a*c)*Sqrt[a + x*(b +
 c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(8*c^(3/2)*Sqrt[x
^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.004, size = 146, normalized size = 1.2

$$\frac{1}{8x}\sqrt{cx^4+bx^3+ax^2}\left(4\sqrt{cx^2+bx+a}c^{5/2}x+2\sqrt{cx^2+bx+a}c^{3/2}b+4\ln\left(1/2\frac{2\sqrt{cx^2+bx+a}\sqrt{c}+2cx+b}{\sqrt{c}}\right)ac^2-\ln\left(\frac{1}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x,x)
```

```
[Out] 1/8*(c*x^4+b*x^3+a*x^2)^(1/2)*(4*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x+2*(c*x^2+b*x
+a)^(1/2)*c^(3/2)*b+4*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2
))*a*c^2-ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^2*c)/(c*
x^2+b*x+a)^(1/2)/c^(5/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x, x)
```

Fricas [A] time = 1.66874, size = 502, normalized size = 4.22

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}\left(2\,c^2x+bc\right)}{16\,c^2x},\ \frac{\left(b^2-4\,ac\right)\sqrt{-c}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4
+ b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x), 1/8*((b^2 - 4*a*c)*sqrt(-c)*x*arc
tan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2
 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x, x)
```

Giac [A] time = 1.16641, size = 169, normalized size = 1.42

$$\frac{1}{8}\left(2\,\sqrt{cx^2+bx+a}\left(2\,x+\frac{b}{c}\right)+\frac{\left(b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}\right)\mathrm{sgn}\left(x\right)-\frac{\left(b^2\log\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(c*x^2 + b*x + a)*(2*x + b/c) + (b^2 - 4*a*c)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2))*sgn(x) - 1/8*(b^2*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2*sq
rt(a)*b*sqrt(c))*sgn(x)/c^(3/2)
```

3.33 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$

Optimal. Leaf size=173

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\sqrt{a}\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x - (Sqrt[a]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2
*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] +
 (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.126323, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1921, 1933, 843, 621, 206, 724}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\sqrt{a}\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x - (Sqrt[a]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2
*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] +
 (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx &= \frac{\sqrt{ax^2+bx^3+cx^4}}{x}+\frac{1}{2}\int \frac{2a+bx}{\sqrt{ax^2+bx^3+cx^4}}\,dx \\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{x}+\frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{2a+bx}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{x}+\frac{\left(ax\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(bx\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}}{2\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\left(2ax\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(bx\sqrt{a+bx+\right.}{} \\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\sqrt{a}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}}{2\sqrt{c}\sqrt{ax^2+bx^3}}
\end{aligned}$$

Mathematica [A] time = 0.110016, size = 134, normalized size = 0.77

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}-2\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)] - 2*Sqrt[a]*Sqrt[
c]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + b*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(2*Sqrt[c]*Sqrt[x^2*(a + x*(b +
 c*x))])
```

Maple [A] time = 0.004, size = 126, normalized size = 0.7

$$-\frac{1}{2x}\sqrt{cx^4+bx^3+ax^2}\left(2\sqrt{a}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)\sqrt{c}-2\sqrt{cx^2+bx+a}\sqrt{c}-b\ln\left(\frac{1}{2}\left(2\sqrt{cx^2+bx+a}\sqrt{c}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(c*x^4+b*x^3+a*x^2)^(1/2)*(2*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+
a)^(1/2))/x)*c^(1/2)-2*(c*x^2+b*x+a)^(1/2)*c^(1/2)-b*ln(1/2*(2*(c*x^2+b*x+a
)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2)))/x/(c*x^2+b*x+a)^(1/2)/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^4+bx^3+ax^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2, x)
```

Fricas [A] time = 1.84208, size = 1457, normalized size = 8.42

$$\left[\frac{b\sqrt{c}x\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c}+\left(b^2+4ac\right)x}{x}\right)+2\sqrt{a}cx\log\left(-\frac{8abx^2+\left(b^2+4ac\right)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{x^3}\right)+4}{4cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(8*a*b*x^
2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)
*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), -1/2*(b*sqrt(-c)*x
*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c
*x^2 + a*c*x)) - sqrt(a)*c*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
- 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 2*sqrt(c*x^4 +
b*x^3 + a*x^2)*c)/(c*x), 1/4*(4*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + b*sqrt(c)*x*lo
g(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(
c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), 1/2*(2*s
qrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*
c*x^3 + a*b*x^2 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*c)/(c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.34 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$

Optimal. Leaf size=173

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3
+ c*x^4]) + (Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]
```

Rubi [A] time = 0.124074, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1920, 1933, 843, 621, 206, 724}

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```


```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3
+ c*x^4]) + (Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]
```


Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} + \frac{1}{2}\int \frac{b+2cx}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} + \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} + \frac{\left(bx\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(cx\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}} \cdots}{\sqrt{ax^2+bx^3+cx^4}}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} - \frac{\left(bx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(2cx\sqrt{a+bx \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} - \frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1} \cdots}{\sqrt{ax^2+bx^3+ \cdots}}$$

Mathematica [A] time = 0.127059, size = 131, normalized size = 0.76

$$-\frac{\sqrt{a+x(b+cx)}\left(bx\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2\sqrt{a}\left(\sqrt{a+x(b+cx)}-\sqrt{c}x\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\right)}{2\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[a + x*(b + c*x)]*(b*x*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b +
 c*x)])] + 2*Sqrt[a]*(Sqrt[a + x*(b + c*x)] - Sqrt[c]*x*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/(2*Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x)
)])
```

Maple [A] time = 0.006, size = 174, normalized size = 1.

$$\frac{1}{2\,a x^{2}}\sqrt{c x^{4}+b x^{3}+a x^{2}}\left(2\,c^{5/2}\sqrt{c x^{2}+b x+a}\,x^{2}-c^{\frac{3}{2}}\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+b x+2\,\sqrt{a}\sqrt{c x^{2}+b x+a}\right)\right)x b-2\,\left(c x^{2}+b x+a\right)^{3/2}c^{3/}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x)
```

```
[Out] 1/2*(c*x^4+b*x^3+a*x^2)^(1/2)*(2*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^2-c^(3/2)*a^
(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x*b-2*(c*x^2+b*x+a)^(3/
2)*c^(3/2)+2*c^(3/2)*(c*x^2+b*x+a)^(1/2)*x*b+2*ln(1/2*(2*(c*x^2+b*x+a)^(1/2
)*c^(1/2)+2*c*x+b)/c^(1/2))*x*a*c^2)/x^2/(c*x^2+b*x+a)^(1/2)/a/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c x^{4}+b x^{3}+a x^{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^3, x)
```

Fricas [A] time = 1.96342, size = 1489, normalized size = 8.61

$$\left[\frac{2\,a\sqrt{c}x^{2}\log\left(-\frac{8\,c^{2}x^{3}+8\,b c x^{2}+4\,\sqrt{c x^{4}+b x^{3}+a x^{2}}(2\,c x+b)\sqrt{c}+\left(b^{2}+4\,a c\right)x}{x}\right)+\sqrt{a}b x^{2}\log\left(-\frac{8\,a b x^{2}+\left(b^{2}+4\,a c\right)x^{3}+8\,a^{2}x-4\,\sqrt{c x^{4}+b x^{3}+a x^{2}}(b x+2\,a)\sqrt{a}}{x^{3}}\right)-}{4\,a x^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 +
a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + sqrt(a)*b*x^2*log(-(8*a*
b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x +
2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), -1/4*(4*a*sq
rt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2
*x^3 + b*c*x^2 + a*c*x)) - sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^
3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*s
qrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(
c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) +
a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)
/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x +
 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*a*sqrt(-c)*x^2*arctan(1/2*s
qrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)
) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (a + bx + cx^2)}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.35 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4} \, dx$

Optimal. Leaf size=114

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a
*x^2) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(8*a^(3/2))
```

Rubi [A] time = 0.147701, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1920, 1951, 12, 1904, 206}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a
*x^2) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(8*a^(3/2))
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}+\frac{1}{4}\int \frac{b+2cx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\int \frac{b^2-4ac}{2\sqrt{ax^2+bx^3+cx^4}}\,dx}{4a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{4a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.100333, size = 112, normalized size = 0.98

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(x^2\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}\right)}{8a^{3/2}x^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)]
+ (b^2 - 4*a*c)*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])
)/(8*a^(3/2)*x^3*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.006, size = 207, normalized size = 1.8

$$-\frac{1}{8\,a^2x^3}\sqrt{cx^4+bx^3+ax^2}\left(4\,ca^{3/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^2+2\,c\sqrt{cx^2+bx+a}x^3b-4\,c\sqrt{cx^2+bx+a}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/8*(c*x^4+b*x^3+a*x^2)^(1/2)*(4*c*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)*x^2+2*c*(c*x^2+b*x+a)^(1/2)*x^3*b-4*c*(c*x^2+b*x+a)^(1/2)*x^
2*a-a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^2*b^2-2*(c*x^2+
```

```
b*x+a)^(3/2)*x*b+2*(c*x^2+b*x+a)^(1/2)*x^2*b^2+4*(c*x^2+b*x+a)^(3/2)*a)/x^3
/(c*x^2+b*x+a)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4, x)
```

Fricas [A] time = 1.69192, size = 517, normalized size = 4.54

$$\left[-\frac{\left(b^2-4\,ac\right)\sqrt{a}x^3\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)}{16\,a^2x^3}, -\frac{\left(b^2-4\,ac\right)\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a
^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/8*((b^2 - 4*a*c)*sqrt(-a
)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3
+ a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x
^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(a + bx + cx^2\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**4, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")

[Out] Timed out
```

3.36 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$

Optimal. Leaf size=155

$$\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*
a*x^3) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^2*x^2) - (b*(b
^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])
])/(16*a^(5/2))
```

Rubi [A] time = 0.255838, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1920, 1951, 12, 1904, 206}

$$\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*
a*x^3) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^2*x^2) - (b*(b
^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])
])/(16*a^(5/2))
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}+\frac{1}{6}\int \frac{b+2cx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}-\frac{\int \frac{\frac{1}{2}\left(3b^2-8ac\right)+bcx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{12a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}+\frac{\int \frac{3b\left(b^2-4ac\right)}{4\sqrt{ax^2+bx^3+c}}}{12a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}+\frac{\left(b\left(b^2-4ac\right)\right)}{}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{\left(b\left(b^2-4ac\right)\right)}{}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b\left(b^2-4ac\right)\mathrm{t}}{}$$

Mathematica [A] time = 0.143892, size = 131, normalized size = 0.85

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(-2\sqrt{a}\sqrt{a+x(b+cx)}\left(8a^2+2ax(b+4cx)-3b^2x^2\right)-3bx^3\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\right)}{48a^{5/2}x^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(8*a^2 - 3*b
^2*x^2 + 2*a*x*(b + 4*c*x)) - 3*b*(b^2 - 4*a*c)*x^3*ArcTanh[(2*a + b*x)/(2*
Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(48*a^(5/2)*x^4*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.006, size = 234, normalized size = 1.5

$$\frac{1}{48\,x^4a^3}\sqrt{cx^4+bx^3+ax^2}\left(12\,ca^{3/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^3b+6\,c\sqrt{cx^2+bx+a}x^4b^2-12\,c\sqrt{cx^2+bx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(1/2)*(12*c*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b
*x+a)^(1/2))/x)*x^3*b+6*c*(c*x^2+b*x+a)^(1/2)*x^4*b^2-12*c*(c*x^2+b*x+a)^(1
/2)*x^3*a*b-3*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^3*b^3
-6*(c*x^2+b*x+a)^(3/2)*x^2*b^2+6*(c*x^2+b*x+a)^(1/2)*x^3*b^3+12*(c*x^2+b*x+
a)^(3/2)*x*a*b-16*(c*x^2+b*x+a)^(3/2)*a^2)/x^4/(c*x^2+b*x+a)^(1/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5, x)
```

Fricas [A] time = 1.84133, size = 609, normalized size = 3.93

$$\left[-\frac{3\left(b^3 - 4\,abc\right)\sqrt{a}x^4 \log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right) + 4\,\sqrt{cx^4 + bx^3 + ax^2}\left(2\,a^2bx + 8\,a^3 - \left(3\,ab^2 - 8\right.\right.}{96\,a^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 +
 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3
*x^4), 1/48*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 +
b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(a + bx + cx^2\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**5, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.37 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$

Optimal. Leaf size=205

$$-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}+\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}$$

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(4*x^5) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*
a*x^4) + ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*a^2*x^3) - (b*(
15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*a^3*x^2) + ((b^2 - 4*a*c
)*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4])])/(128*a^(7/2))
```

Rubi [A] time = 0.385848, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1920, 1951, 12, 1904, 206}

$$-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}+\frac{\left(b^2-4ac\right)\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(4*x^5) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*
a*x^4) + ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*a^2*x^3) - (b*(
15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*a^3*x^2) + ((b^2 - 4*a*c
)*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4])])/(128*a^(7/2))
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}+\frac{1}{8}\int \frac{b+2cx}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}-\frac{\int \frac{\frac{1}{2}\left(5b^2-12ac\right)+2bcx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{24a} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}+\int \frac{\frac{1}{4}b\left(15b^2-52a\right.}{x\sqrt{ax}} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b\left(15b^2-52\right.}{} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b\left(15b^2-52\right.}{} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b\left(15b^2-52\right.}{} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b\left(15b^2-52\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.225157, size = 160, normalized size = 0.78

$$
\frac{\sqrt{x^2(a+x(b+cx))}\left(3x^4\left(16a^2c^2-24ab^2c+5b^4\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}\sqrt{a+x(b+cx)}\left(8a^2x(b+3cx)+48a^3\right.\right.}{384a^{7/2}x^5\sqrt{a+x(b+cx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(48*a^3 + 15
*b^3*x^3 + 8*a^2*x*(b + 3*c*x) - 2*a*b*x^2*(5*b + 26*c*x)) + 3*(5*b^4 - 24*
a*b^2*c + 16*a^2*c^2)*x^4*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*
x)])]))/(384*a^(7/2)*x^5*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.006, size = 387, normalized size = 1.9

$$\frac{1}{384\,x^5a^4}\sqrt{cx^4+bx^3+ax^2}\left(48\,c^2a^{5/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^4+24\,c^2\sqrt{cx^2+bx+a}\,x^5ab-72\,ca^{3/2}\ln\left(\frac{2\,a+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x)
```

```
[Out] 1/384*(c*x^4+b*x^3+a*x^2)^(1/2)*(48*c^2*a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^
2+b*x+a)^(1/2))/x)*x^4+24*c^2*(c*x^2+b*x+a)^(1/2)*x^5*a*b-72*c*a^(3/2)*ln((
2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^4*b^2-48*c^2*(c*x^2+b*x+a)^(1/2
)*x^4*a^2-30*c*(c*x^2+b*x+a)^(1/2)*x^5*b^3-24*c*(c*x^2+b*x+a)^(3/2)*x^3*a*b
+84*c*(c*x^2+b*x+a)^(1/2)*x^4*a*b^2+15*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2
+b*x+a)^(1/2))/x)*x^4*b^4+48*c*(c*x^2+b*x+a)^(3/2)*x^2*a^2+30*(c*x^2+b*x+a)
^(3/2)*x^3*b^3-30*(c*x^2+b*x+a)^(1/2)*x^4*b^4-60*(c*x^2+b*x+a)^(3/2)*x^2*a*
b^2+80*(c*x^2+b*x+a)^(3/2)*x*a^2*b-96*(c*x^2+b*x+a)^(3/2)*a^3)/x^5/(c*x^2+b
*x+a)^(1/2)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{cx^4+bx^3+ax^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6, x)
```

Fricas [A] time = 2.12879, size = 757, normalized size = 3.69

$$\left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{a}x^5\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\left(8\,a^3bx+48\,a^4+\left(15\,ab^3-52\,a^2\right.\right.}{768\,a^4x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (
b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt
(a))/x^3) - 4*(8*a^3*b*x + 48*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*
b^2 - 12*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^5), -1/384*(3*(5*b
^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 +
a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(8*a^3*b*x + 4
8*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^2)*sqrt(c*
x^4 + b*x^3 + a*x^2))/(a^4*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**6, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.38 $\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=422

$$\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{35840c^3}-\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+bx^3+cx^4}}{71680c^4}+\frac{\left(18896a^2b^2c^2\right.}{}$$

```
[Out] ((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 - 6720*a^3*c^3)*Sqrt[a*x^2 +
b*x^3 + c*x^4])/(286720*c^5) - (b*(3465*b^6 - 30660*a*b^4*c + 81648*a^2*b^2
*c^2 - 58816*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(573440*c^6*x) - (b*(231
*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(71680*c
^4) + ((99*b^4 - 568*a*b^2*c + 560*a^2*c^2)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]
)/(35840*c^3) - (x^3*(b*(11*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*Sqrt[
a*x^2 + b*x^3 + c*x^4])/(4480*c^2) + (x*(3*b + 14*c*x)*(a*x^2 + b*x^3 + c*x
^4)^(3/2))/(112*c) + (3*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*
x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(32768*c^(13/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 1.20339, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1919, 1945, 1949, 12, 1914, 621, 206}

$$\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{35840c^3}-\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+bx^3+cx^4}}{71680c^4}+\frac{\left(18896a^2b^2c^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] ((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 - 6720*a^3*c^3)*Sqrt[a*x^2 +
b*x^3 + c*x^4])/(286720*c^5) - (b*(3465*b^6 - 30660*a*b^4*c + 81648*a^2*b^2
*c^2 - 58816*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(573440*c^6*x) - (b*(231
*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(71680*c
^4) + ((99*b^4 - 568*a*b^2*c + 560*a^2*c^2)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]
)/(35840*c^3) - (x^3*(b*(11*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*Sqrt[
a*x^2 + b*x^3 + c*x^4])/(4480*c^2) + (x*(3*b + 14*c*x)*(a*x^2 + b*x^3 + c*x
^4)^(3/2))/(112*c) + (3*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*
x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(32768*c^(13/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}dx=\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}+\frac{3\int x^2\left(-4ab-\frac{1}{2}\left(11b^2-28ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}\,d}{112c}$$

$$=-\frac{x^3\left(b\left(11b^2+68ac\right)+10c\left(11b^2-28ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}}{4480c^2}+\frac{x(3b+14cx)\left(ax^2+bx^3\right.}{112c}$$

$$=\frac{\left(99b^4-568ab^2c+560a^2c^2\right)x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3}-\frac{x^3\left(b\left(11b^2+68ac\right)+10c\left(11b^2-28ac\right.\right.}{4480c^2}$$

$$=-\frac{b\left(231b^4-1560ab^2c+2416a^2c^2\right)x\sqrt{ax^2+bx^3+cx^4}}{71680c^4}+\frac{\left(99b^4-568ab^2c+560a^2c^2\right)x^2\sqrt{}}{35840c^3}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(231b^4-1560ab^2c\right.}{}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(3465b^6-30660ab\right.}{}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(3465b^6-30660ab\right.}{}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(3465b^6-30660ab\right.}{}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(3465b^6-30660ab\right.}{}$$

$$=\frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{b\left(3465b^6-30660ab\right.}{}$$

Mathematica [A] time = 0.426287, size = 236, normalized size = 0.56

$$\frac{\left(x^2(a+x(b+cx))\right)^{3/2}\left(\frac{\left(16a^2c^2-72ab^2c+33b^4\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{4096c^{11/2}x^3(a+x(b+cx))^{3/2}}+\frac{(372abc-280}{}\right.}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*(a + b*x + c*x^2 - (11*b*(a + x*(b + c*x)))/
(14*c*x) + ((-231*b^3 + 372*a*b*c + 330*b^2*c*x - 280*a*c^2*x)*(a + x*(b +
c*x)))/(560*c^3*x^3) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(
b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(40
96*c^(11/2)*x^3*(a + x*(b + c*x))^(3/2))))/(8*c)
```

Maple [A] time = 0.008, size = 649, normalized size = 1.5

$$\frac{1}{1146880\,x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(26880\ln\left(1/2\,\frac{2\sqrt{cx^2+bx+a}\sqrt{c}+2\,cx+b}{\sqrt{c}}\right)a^4c^5+3465\ln\left(1/2\,\frac{2\sqrt{cx^2+bx+a}\sqrt{c}+2}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/1146880*(c*x^4+b*x^3+a*x^2)^(3/2)*(26880*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^
(1/2)+2*c*x+b)/c^(1/2))*a^4*c^5+3465*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+
2*c*x+b)/c^(1/2))*b^8*c+143360*x^3*(c*x^2+b*x+a)^(5/2)*c^(13/2)-59136*(c*x^
2+b*x+a)^(5/2)*c^(7/2)*b^3+18480*(c*x^2+b*x+a)^(3/2)*c^(5/2)*b^5-6930*(c*x^
2+b*x+a)^(1/2)*c^(3/2)*b^7-112640*(c*x^2+b*x+a)^(5/2)*c^(11/2)*x^2*b-71680*
(c*x^2+b*x+a)^(5/2)*c^(11/2)*x*a+84480*(c*x^2+b*x+a)^(5/2)*c^(9/2)*x*b^2+95
232*(c*x^2+b*x+a)^(5/2)*c^(9/2)*a*b+17920*(c*x^2+b*x+a)^(3/2)*c^(11/2)*x*a^
2-134400*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^3*b^2*c^
4+117600*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^4*c^
3-35280*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^6*c^2-4
0320*(c*x^2+b*x+a)^(3/2)*c^(7/2)*a*b^3+26880*(c*x^2+b*x+a)^(1/2)*c^(11/2)*x
*a^3-13860*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b^6+13440*(c*x^2+b*x+a)^(1/2)*c^(9
/2)*a^3*b-63840*(c*x^2+b*x+a)^(1/2)*c^(7/2)*a^2*b^3+42840*(c*x^2+b*x+a)^(1/
2)*c^(5/2)*a*b^5+36960*(c*x^2+b*x+a)^(3/2)*c^(7/2)*x*b^4+8960*(c*x^2+b*x+a)
^(3/2)*c^(9/2)*a^2*b-80640*(c*x^2+b*x+a)^(3/2)*c^(9/2)*x*a*b^2-127680*(c*x^
2+b*x+a)^(1/2)*c^(9/2)*x*a^2*b^2+85680*(c*x^2+b*x+a)^(1/2)*c^(7/2)*x*a*b^4)
/x^3/(c*x^2+b*x+a)^(3/2)/c^(15/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x, x)
```

Fricas [A] time = 2.03112, size = 1600, normalized size = 3.79

$$\left[\frac{105\left(33\,b^8 - 336\,ab^6c + 1120\,a^2b^4c^2 - 1280\,a^3b^2c^3 + 256\,a^4c^4\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3 + 8\,bcx^2 + 4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c} + \left(b^2+4\,ac\right)x}{x}\right)}{\quad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2293760*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
 + 256*a^4*c^4)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(71680*c^8*x^7 + 8
7040*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a
^3*b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^
4 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 15
60*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 188
96*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x), -1/
1146880*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 +
256*a^4*c^4)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
```

```
sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(71680*c^8*x^7 + 87040*b*c^7*x^6
- 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 + 1280
*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4 + 16*(99*b^4*
c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 +
2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4
- 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A] time = 1.29441, size = 703, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/573440*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*c*x*sgn(x) + 17*b*sgn
(x))*x + (b^2*c^6*sgn(x) + 84*a*c^7*sgn(x))/c^7)*x - (11*b^3*c^5*sgn(x) - 5
2*a*b*c^6*sgn(x))/c^7)*x + (99*b^4*c^4*sgn(x) - 568*a*b^2*c^5*sgn(x) + 560*
a^2*c^6*sgn(x))/c^7)*x - (231*b^5*c^3*sgn(x) - 1560*a*b^3*c^4*sgn(x) + 2416
*a^2*b*c^5*sgn(x))/c^7)*x + (1155*b^6*c^2*sgn(x) - 8988*a*b^4*c^3*sgn(x) +
18896*a^2*b^2*c^4*sgn(x) - 6720*a^3*c^5*sgn(x))/c^7)*x - (3465*b^7*c*sgn(x)
 - 30660*a*b^5*c^2*sgn(x) + 81648*a^2*b^3*c^3*sgn(x) - 58816*a^3*b*c^4*sgn(
x))/c^7) - 3/32768*(33*b^8*sgn(x) - 336*a*b^6*c*sgn(x) + 1120*a^2*b^4*c^2*s
gn(x) - 1280*a^3*b^2*c^3*sgn(x) + 256*a^4*c^4*sgn(x))*log(abs(-2*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2) + 1/1146880*(3465*b^8*log(
abs(-b + 2*sqrt(a)*sqrt(c))) - 35280*a*b^6*c*log(abs(-b + 2*sqrt(a)*sqrt(c)
)) + 117600*a^2*b^4*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 134400*a^3*b^2*c
^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 26880*a^4*c^4*log(abs(-b + 2*sqrt(a)*
sqrt(c))) + 6930*sqrt(a)*b^7*sqrt(c) - 61320*a^(3/2)*b^5*c^(3/2) + 163296*a
^(5/2)*b^3*c^(5/2) - 117632*a^(7/2)*b*c^(7/2))*sgn(x)/c^(13/2)
```

3.39 $\int \left(ax^2+bx^3+cx^4\right)^{3/2} dx$

Optimal. Leaf size=364

$$-\frac{b\left(1168a^2c^2-728ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{\left(5488a^2b^2c^2-2048a^3c^3-2520ab^4c+315b^6\right)\sqrt{ax^2+bx^3+cx}}{35840c^5x}$$

```
[Out] -(b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(17
920*c^4) + ((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(35840*c^5*x) + ((7*b^2 - 32*a*c)*(3*b^2 - 4*a*c)*
x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4480*c^3) - (b*(9*b^2 - 44*a*c)*x^2*Sqrt[a*
x^2 + b*x^3 + c*x^4])/(2240*c^2) + (x^3*(b^2 + 24*a*c + 10*b*c*x)*Sqrt[a*x^
2 + b*x^3 + c*x^4])/(280*c) + (x*(a*x^2 + b*x^3 + c*x^4)^(3/2))/7 - (3*b*(b
^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2)*Sqrt[a*x^2 + b*x^3 + c*x^
4])
```

Rubi [A] time = 1.03943, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1906, 1945, 1949, 12, 1914, 621, 206}

$$-\frac{b\left(1168a^2c^2-728ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{\left(5488a^2b^2c^2-2048a^3c^3-2520ab^4c+315b^6\right)\sqrt{ax^2+bx^3+cx}}{35840c^5x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```


```
[Out] -(b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(17
920*c^4) + ((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(35840*c^5*x) + ((7*b^2 - 32*a*c)*(3*b^2 - 4*a*c)*
x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4480*c^3) - (b*(9*b^2 - 44*a*c)*x^2*Sqrt[a*
x^2 + b*x^3 + c*x^4])/(2240*c^2) + (x^3*(b^2 + 24*a*c + 10*b*c*x)*Sqrt[a*x^
2 + b*x^3 + c*x^4])/(280*c) + (x*(a*x^2 + b*x^3 + c*x^4)^(3/2))/7 - (3*b*(b
^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2)*Sqrt[a*x^2 + b*x^3 + c*x^
4])
```


Rule 1906

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[(x*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(p*(2*n - q) + 1), x] + Dis
t[((n - q)*p)/(p*(2*n - q) + 1), Int[x^q*(2*a + b*x^(n - q))*(a*x^q + b*x^n
 + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*
n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0]
&& NeQ[p*(2*n - q) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
```

```
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2} dx = \frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2} + \frac{3}{14}\int x^2(2a+bx)\sqrt{ax^2+bx^3+cx^4}\,dx$$

$$= \frac{x^3\left(b^2+24ac+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{280c} + \frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2} + \frac{\int \frac{x^4\left(-4a\left(b^2-6ac\right)-\frac{1}{2}b\right.}{\sqrt{ax^2+bx^3}}}{280c}$$

$$= -\frac{b\left(9b^2-44ac\right)x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2} + \frac{x^3\left(b^2+24ac+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{280c} + \frac{1}{7}x\left(\right.$$

$$= \frac{\left(7b^2-32ac\right)\left(3b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}}{4480c^3} - \frac{b\left(9b^2-44ac\right)x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2} + \frac{x^3\left(\right.}{}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(7b^2-32ac\right)\left(3b^2-4ac\right)x\sqrt{ax^2+}}{4480c^3}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(315b^6-2520ab^4c+5488a^2b^2c^2\right.}{35840}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(315b^6-2520ab^4c+5488a^2b^2c^2\right.}{35840}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(315b^6-2520ab^4c+5488a^2b^2c^2\right.}{35840}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(315b^6-2520ab^4c+5488a^2b^2c^2\right.}{35840}$$

$$= -\frac{b\left(105b^4-728ab^2c+1168a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4} + \frac{\left(315b^6-2520ab^4c+5488a^2b^2c^2\right.}{35840}$$

Mathematica [A] time = 0.267093, size = 197, normalized size = 0.54

$$\frac{\left(x^2(a+x(b+cx))\right)^{3/2}\left(\frac{7\left(4abc-3b^3\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2048c^{9/2}(a+x(b+cx))^{3/2}} + \frac{\left(-16ac+21b^2-30bc\right.}{40c^2}\right.}{7cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*(x^2*(a + x*(b + c*x)) + ((21*b^2 - 16*a*c -
 30*b*c*x)*(a + x*(b + c*x)))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*Sqrt[c]*(
b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) +
 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))
/(2048*c^(9/2)*(a + x*(b + c*x))^(3/2))))/(7*c*x^3)
```

Maple [A] time = 0.009, size = 479, normalized size = 1.3

$$\frac{1}{71680\,x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(10240\,x^2\left(cx^2+bx+a\right)^{5/2}c^{11/2}-7680\left(cx^2+bx+a\right)^{5/2}c^{9/2}xb-4096\left(cx^2+bx+a\right)^{5/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/71680*(c*x^4+b*x^3+a*x^2)^(3/2)*(10240*x^2*(c*x^2+b*x+a)^(5/2)*c^(11/2)-7
680*(c*x^2+b*x+a)^(5/2)*c^(9/2)*x*b-4096*(c*x^2+b*x+a)^(5/2)*c^(9/2)*a+5376
*(c*x^2+b*x+a)^(5/2)*c^(7/2)*b^2+4480*(c*x^2+b*x+a)^(3/2)*c^(9/2)*x*a*b-336
0*(c*x^2+b*x+a)^(3/2)*c^(7/2)*x*b^3+2240*(c*x^2+b*x+a)^(3/2)*c^(7/2)*a*b^2-
1680*(c*x^2+b*x+a)^(3/2)*c^(5/2)*b^4+6720*(c*x^2+b*x+a)^(1/2)*c^(9/2)*x*a^2
*b-6720*(c*x^2+b*x+a)^(1/2)*c^(7/2)*x*a*b^3+1260*(c*x^2+b*x+a)^(1/2)*c^(5/2
)*x*b^5+3360*(c*x^2+b*x+a)^(1/2)*c^(7/2)*a^2*b^2-3360*(c*x^2+b*x+a)^(1/2)*c
^(5/2)*a*b^4+630*(c*x^2+b*x+a)^(1/2)*c^(3/2)*b^6+6720*ln(1/2*(2*(c*x^2+b*x+
a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^3*b*c^4-8400*ln(1/2*(2*(c*x^2+b*x+a)^(
1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^3*c^3+2940*ln(1/2*(2*(c*x^2+b*x+a)^(1/
2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^5*c^2-315*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^
(1/2)+2*c*x+b)/c^(1/2))*b^7*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A] time = 1.98469, size = 1299, normalized size = 3.57

$$\left[-\frac{105\left(3\,b^7 - 28\,ab^5c + 80\,a^2b^3c^2 - 64\,a^3bc^3\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right) - 4\left(5120\,c^7x^6 + 640\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(5120*c^7*x^6 + 6400*b*c^6*x^5 + 315*b^6
*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 128*(b^2*c^5 + 64*a
*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4
 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x), 1/71680*(105*(3*b^7 - 28*a*b^5*c + 80*
a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(5120*c^7*x^6 + 64
00*b*c^6*x^5 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4
 + 128*(b^2*c^5 + 64*a*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b
^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3
+ 1168*a^2*b*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(3/2), x)
```

Giac [A] time = 1.30208, size = 579, normalized size = 1.59

$$\frac{1}{35840}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,(4\,cx\mathrm{sgn}\,(x)+5\,b\mathrm{sgn}\,(x))x+\frac{b^2c^5\mathrm{sgn}\,(x)+64\,ac^6\mathrm{sgn}\,(x)}{c^6}\right)x-\frac{9\,b^3c^4\mathrm{sgn}\,(x)-44\,}{c^6}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35840*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*c*x*sgn(x) + 5*b*sgn(x))*x
 + (b^2*c^5*sgn(x) + 64*a*c^6*sgn(x))/c^6)*x - (9*b^3*c^4*sgn(x) - 44*a*b*c
^5*sgn(x))/c^6)*x + (21*b^4*c^3*sgn(x) - 124*a*b^2*c^4*sgn(x) + 128*a^2*c^5
*sgn(x))/c^6)*x - (105*b^5*c^2*sgn(x) - 728*a*b^3*c^3*sgn(x) + 1168*a^2*b*c
^4*sgn(x))/c^6)*x + (315*b^6*c*sgn(x) - 2520*a*b^4*c^2*sgn(x) + 5488*a^2*b^
2*c^3*sgn(x) - 2048*a^3*c^4*sgn(x))/c^6) + 3/2048*(3*b^7*sgn(x) - 28*a*b^5*
c*sgn(x) + 80*a^2*b^3*c^2*sgn(x) - 64*a^3*b*c^3*sgn(x))*log(abs(-2*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2) - 1/71680*(315*b^7*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 2940*a*b^5*c*log(abs(-b + 2*sqrt(a)*sqrt(c)))
 + 8400*a^2*b^3*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 6720*a^3*b*c^3*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) + 630*sqrt(a)*b^6*sqrt(c) - 5040*a^(3/2)*b^4*c^
(3/2) + 10976*a^(5/2)*b^2*c^(5/2) - 4096*a^(7/2)*c^(7/2))*sgn(x)/c^(11/2)
```

3.40 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=288

$$\frac{(240a^2c^2-216ab^2c+35b^4)\sqrt{ax^2+bx^3+cx^4}}{3840c^3}-\frac{b(1296a^2c^2-760ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x}-\frac{x(6cx(7b^2-20ac$$

```
[Out] ((35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3840*c^
3) - (b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(7680*c^4*x) - (x*(b*(7*b^2 + 12*a*c) + 6*c*(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2
 + b*x^3 + c*x^4])/(960*c^2) + ((3*b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2
))/(60*c*x) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.519235, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1919, 1934, 1949, 12, 1914, 621, 206}

$$\frac{(240a^2c^2-216ab^2c+35b^4)\sqrt{ax^2+bx^3+cx^4}}{3840c^3}-\frac{b(1296a^2c^2-760ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x}-\frac{x(6cx(7b^2-20ac$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```


```
[Out] ((35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3840*c^
3) - (b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(7680*c^4*x) - (x*(b*(7*b^2 + 12*a*c) + 6*c*(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2
 + b*x^3 + c*x^4])/(960*c^2) + ((3*b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2
))/(60*c*x) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])
```


Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1934

```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_)*((A_) + (
B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x*(b*B*(n - q)*p + A*c*(p*q + (n - q)*
(2*p + 1) + 1) + B*c*(p*(2*n - q) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
```

```
 - q))^p)/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((
n - q)*p)/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1)), Int[x^q*(2*a
*A*c*(p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(p*q + 1) + (2*a*B*c*(p*(2*n - q
) + 1) + A*b*c*(p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(p*q + (n - q)*p + 1))
*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b,
 c, A, B, n, q}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] &&
 NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p*(2*n - q) + 1, 0] && NeQ[p*q + (
n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x}\,dx = \frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx} + \frac{\int \left(-2ab+\frac{1}{2}\left(-7b^2+20ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}\,dx}{20c}$$

$$= -\frac{x\left(b\left(7b^2+12ac\right)+6c\left(7b^2-20ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}}{960c^2} + \frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3}}{60cx}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{x\left(b\left(7b^2+12ac\right)+6c\left(7b^2-20ac\right)x\right)\sqrt{ax}}{960c^2}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right)\sqrt{ax^2+b}}{7680c^4x}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right)\sqrt{ax^2+b}}{7680c^4x}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right)\sqrt{ax^2+b}}{7680c^4x}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right)\sqrt{ax^2+b}}{7680c^4x}$$

$$= \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b\left(105b^4-760ab^2c+1296a^2c^2\right)\sqrt{ax^2+b}}{7680c^4x}$$

Mathematica [A] time = 0.242008, size = 180, normalized size = 0.62

$$\frac{\left(x^2(a+x(b+cx))\right)^{3/2}\left(\frac{\left(7b^2-4ac\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{512c^{7/2}(a+x(b+cx))^{3/2}} + x(a+x(b+cx)) - 7\right.}{6cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*((-7*b*(a + x*(b + c*x)))/(10*c) + x*(a + x*
(b + c*x)) + ((7*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*
(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(512*c^(7/2)*(a + x*(b + c*x))^(
3/2))))/(6*c*x^3)
```

Maple [A] time = 0.009, size = 431, normalized size = 1.5

$$\frac{1}{15360\,x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(2560\,x\left(cx^2+bx+a\right)^{5/2}c^{9/2}-1792\,c^{7/2}\left(cx^2+bx+a\right)^{5/2}b-640\,c^{9/2}\left(cx^2+bx+a\right)^{3/2}xa+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x)
```

```
[Out] 1/15360*(c*x^4+b*x^3+a*x^2)^(3/2)*(2560*x*(c*x^2+b*x+a)^(5/2)*c^(9/2)-1792*
c^(7/2)*(c*x^2+b*x+a)^(5/2)*b-640*c^(9/2)*(c*x^2+b*x+a)^(3/2)*x*a+1120*c^(7
```

```
/2)*(c*x^2+b*x+a)^(3/2)*x*b^2-320*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a*b+560*c^(5/
2)*(c*x^2+b*x+a)^(3/2)*b^3-960*c^(9/2)*(c*x^2+b*x+a)^(1/2)*x*a^2+1920*c^(7/
2)*(c*x^2+b*x+a)^(1/2)*x*a*b^2-420*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x*b^4-480*c^
(7/2)*(c*x^2+b*x+a)^(1/2)*a^2*b+960*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b^3-210*c
^(3/2)*(c*x^2+b*x+a)^(1/2)*b^5-960*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*
c*x+b)/c^(1/2))*a^3*c^4+2160*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)
/c^(1/2))*a^2*b^2*c^3-900*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^
(1/2))*a*b^4*c^2+105*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2)
)*b^6*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x, x)
```

Fricas [A] time = 1.83513, size = 1099, normalized size = 3.82

$$\left[-\frac{15\left(7b^6 - 60ab^4c + 144a^2b^2c^2 - 64a^3c^3\right)\sqrt{c}x\log\left(-\frac{8c^2x^3+8bcx^2-4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c}+\left(b^2+4ac\right)x}{x}\right) - 4\left(1280c^6x^5 + 16\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*x
*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sq
rt(c) + (b^2 + 4*a*c)*x)/x) - 4*(1280*c^6*x^5 + 1664*b*c^5*x^4 - 105*b^5*c
+ 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^
3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/15360*(15*(7*b^6 - 60*a*b^4*c + 144
*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(1280*c^6*x^5 + 166
4*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 +
140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2
*c^3 + 240*a^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x, x)
```

Giac [A] time = 1.32493, size = 493, normalized size = 1.71

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,(10\,cx\mathrm{sgn}\,(x)+13\,b\mathrm{sgn}\,(x))x+\frac{3\,b^2c^4\mathrm{sgn}\,(x)+140\,ac^5\mathrm{sgn}\,(x)}{c^5}\right)x-\frac{7\,b^3c^3\mathrm{sgn}\,(x)-36\,abc}{c^5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*x*sgn(x) + 13*b*sgn(x))*x +
(3*b^2*c^4*sgn(x) + 140*a*c^5*sgn(x))/c^5)*x - (7*b^3*c^3*sgn(x) - 36*a*b*c
^4*sgn(x))/c^5)*x + (35*b^4*c^2*sgn(x) - 216*a*b^2*c^3*sgn(x) + 240*a^2*c^4
*sgn(x))/c^5)*x - (105*b^5*c*sgn(x) - 760*a*b^3*c^2*sgn(x) + 1296*a^2*b*c^3
*sgn(x))/c^5) - 1/1024*(7*b^6*sgn(x) - 60*a*b^4*c*sgn(x) + 144*a^2*b^2*c^2*
sgn(x) - 64*a^3*c^3*sgn(x))*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
sqrt(c) - b))/c^(9/2) + 1/15360*(105*b^6*log(abs(-b + 2*sqrt(a)*sqrt(c))) -
 900*a*b^4*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2160*a^2*b^2*c^2*log(abs(-b
 + 2*sqrt(a)*sqrt(c))) - 960*a^3*c^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 210
*sqrt(a)*b^5*sqrt(c) - 1520*a^(3/2)*b^3*c^(3/2) + 2592*a^(5/2)*b*c^(5/2))*s
gn(x)/c^(9/2)
```

3.41 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=198

$$\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{3bx\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{b(b+2cx)\left(ax^2+\right.}{16c^2x}$$

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(128*c^3*x) - (
b*(b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(16*c^2*x^3) + (a*x^2 + b*x^3
+ c*x^4)^(5/2)/(5*c*x^5) - (3*b*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.178102, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1917, 1918, 1914, 621, 206}

$$\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{3bx\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{b(b+2cx)\left(ax^2+\right.}{16c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(128*c^3*x) - (
b*(b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(16*c^2*x^3) + (a*x^2 + b*x^3
+ c*x^4)^(5/2)/(5*c*x^5) - (3*b*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])
```

Rule 1917

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n)*(a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
)/(2*c*(p + 1)), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*
x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p,
 q] && EqQ[m + p*(n - 1) - 1, 0]
```

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
```

```
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^2}\,dx &= \frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-\frac{b\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^3}\,dx}{2c}\\
&= -\frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3}+\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}+\frac{\left(3b\left(b^2-4ac\right)\right)\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx}{32c^2}\\
&= \frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3}+\frac{\left(ax^2+bx^3+\ldots\right.}{5cx^5}\\
&= \frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3}+\frac{\left(ax^2+bx^3+\ldots\right.}{5cx^5}\\
&= \frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3}+\frac{\left(ax^2+bx^3+\ldots\right.}{5cx^5}\\
&= \frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3}+\frac{\left(ax^2+bx^3+\ldots\right.}{5cx^5}\end{aligned}$$

Mathematica [A] time = 0.182453, size = 163, normalized size = 0.82

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(4b^2c\left(2cx^2-25a\right)+8bc^2x\left(7a+22cx^2\right)+128c^2\left(a+cx^2\right)^2-10b^3cx+15b^4\right)-15b\ldots\right.}{1280c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^4 - 10*b^3*
c*x + 128*c^2*(a + c*x^2)^2 + 4*b^2*c*(-25*a + 2*c*x^2) + 8*b*c^2*x*(7*a +
22*c*x^2)) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])]))/(1280*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.006, size = 289, normalized size = 1.5

$$\frac{1}{1280\,x^{3}}\left(cx^{4}+bx^{3}+ax^{2}\right)^{\frac{3}{2}}\left(256\left(cx^{2}+bx+a\right)^{5/2}c^{7/2}-160\,c^{7/2}\left(cx^{2}+bx+a\right)^{3/2}xb-80\,c^{5/2}\left(cx^{2}+bx+a\right)^{3/2}b^{2}-24\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x)
```

```
[Out] 1/1280*(c*x^4+b*x^3+a*x^2)^(3/2)*(256*(c*x^2+b*x+a)^(5/2)*c^(7/2)-160*c^(7/
2)*(c*x^2+b*x+a)^(3/2)*x*b-80*c^(5/2)*(c*x^2+b*x+a)^(3/2)*b^2-240*c^(7/2)*(
c*x^2+b*x+a)^(1/2)*x*a*b+60*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x*b^3-120*c^(5/2)*(
c*x^2+b*x+a)^(1/2)*a*b^2+30*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^4-240*ln(1/2*(2*(
c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b*c^3+120*ln(1/2*(2*(c*x^2
+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^3*c^2-15*ln(1/2*(2*(c*x^2+b*x+a
)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^5*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{4}+bx^{3}+ax^{2}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2, x)
```

Fricas [A] time = 1.69484, size = 878, normalized size = 4.43

$$\left[\frac{15\left(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2}\right)\sqrt{c}x\log\left(-\frac{8\,c^{2}x^{3}+8\,bcx^{2}-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}(2\,cx+b)\sqrt{c}+\left(b^{2}+4\,ac\right)x}{x}\right)+4\left(128\,c^{5}x^{4}+176\,bc^{4}x^{3}+15\,b^{4}c-\right.}{2560\,c^{4}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*
b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)
*x)/x) + 4*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^
2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt(c*x
^4 + b*x^3 + a*x^2))/(c^4*x), 1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*s
qrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*
x^3 + b*c*x^2 + a*c*x)) + 2*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a
*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b
*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**2, x)
```

Giac [A] time = 1.27319, size = 383, normalized size = 1.93

$$\frac{1}{640}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,cx\mathrm{sgn}\left(x\right)+11\,b\mathrm{sgn}\left(x\right)\right)x+\frac{b^2c^3\mathrm{sgn}\left(x\right)+32\,ac^4\mathrm{sgn}\left(x\right)}{c^4}\right)x-\frac{5\,b^3c^2\mathrm{sgn}\left(x\right)-28\,abc^3\mathrm{sgn}\left(x\right)}{c^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/640*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*c*x*sgn(x) + 11*b*sgn(x))*x + (b^2*
c^3*sgn(x) + 32*a*c^4*sgn(x))/c^4)*x - (5*b^3*c^2*sgn(x) - 28*a*b*c^3*sgn(x
))/c^4)*x + (15*b^4*c*sgn(x) - 100*a*b^2*c^2*sgn(x) + 128*a^2*c^3*sgn(x))/c
^4) + 3/256*(b^5*sgn(x) - 8*a*b^3*c*sgn(x) + 16*a^2*b*c^2*sgn(x))*log(abs(-
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2) - 1/1280*(15*b^
5*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c*log(abs(-b + 2*sqrt(a)*sqr
t(c))) + 240*a^2*b*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^4*sq
rt(c) - 200*a^(3/2)*b^2*c^(3/2) + 256*a^(5/2)*c^(5/2))*sgn(x)/c^(7/2)
```

3.42 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=165

$$-\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x}+\frac{3x(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3}$$

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*c^2*x) + ((b
 + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(8*c*x^3) + (3*(b^2 - 4*a*c)^2*x*S
qrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(128*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.127758, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1918, 1914, 621, 206}

$$-\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x}+\frac{3x(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*c^2*x) + ((b
 + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(8*c*x^3) + (3*(b^2 - 4*a*c)^2*x*S
qrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(128*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^3}\,dx = \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} - \frac{\left(3\left(b^2-4ac\right)\right)\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx}{16c}$$

$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} + \frac{\left(3\left(b^2-4ac\right)^2\right.}{1}$$

$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} + \frac{\left(3\left(b^2-4ac\right)^2\right.}{128}$$

$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} + \frac{\left(3\left(b^2-4ac\right)^2\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3} + \frac{3\left(b^2-4ac\right)^2 x}{1}$$

Mathematica [A] time = 0.0644026, size = 132, normalized size = 0.8

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{128c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b
^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(128*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x
))])
```

Maple [A] time = 0.006, size = 265, normalized size = 1.6

$$\frac{1}{128\,x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(32\,x\left(cx^2+bx+a\right)^{3/2}c^{7/2}+16\,c^{5/2}\left(cx^2+bx+a\right)^{3/2}b+48\,c^{7/2}\sqrt{cx^2+bx+a}\,xa-12\,c^{5/2}\sqrt{cx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x)
```

```
[Out] 1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(32*x*(c*x^2+b*x+a)^(3/2)*c^(7/2)+16*c^(5/2
)*(c*x^2+b*x+a)^(3/2)*b+48*c^(7/2)*(c*x^2+b*x+a)^(1/2)*x*a-12*c^(5/2)*(c*x^
2+b*x+a)^(1/2)*x*b^2+24*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b-6*c^(3/2)*(c*x^2+b*
x+a)^(1/2)*b^3+48*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a
^2*c^3-24*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^2*c^2
+3*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^4*c)/x^3/(c*x^
```

```
2+b*x+a)^(3/2)/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3, x)
```

Fricas [A] time = 1.64564, size = 725, normalized size = 4.39

$$\left[\frac{3\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right) + 4\left(16\,c^4x^3 + 24\,bc^3x^2 - 3\,b^3c + 20\,a\right.}{256\,c^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*
x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/
x) + 4*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*
a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -1/128*(3*(b^4 - 8*a*b^2*c
+ 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b
^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/
(c^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**3, x)
```

Giac [A] time = 1.28001, size = 313, normalized size = 1.9

$$\frac{1}{64}\sqrt{cx^2 + bx + a}\left(2\left(4\left(2\,cx\mathrm{sgn}\left(x\right) + 3\,b\mathrm{sgn}\left(x\right)\right)x + \frac{b^2c^2\mathrm{sgn}\left(x\right) + 20\,ac^3\mathrm{sgn}\left(x\right)}{c^3}\right)x - \frac{3\,b^3c\mathrm{sgn}\left(x\right) - 20\,abc^2\mathrm{sgn}\left(x\right)}{c^3}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c*x*sgn(x) + 3*b*sgn(x))*x + (b^2*c^2*s
gn(x) + 20*a*c^3*sgn(x))/c^3)*x - (3*b^3*c*sgn(x) - 20*a*b*c^2*sgn(x))/c^3)
 - 3/128*(b^4*sgn(x) - 8*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(-2*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) + 1/128*(3*b^4*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 24*a*b^2*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) +
 48*a^2*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^3*sqrt(c) - 40*a
^(3/2)*b*c^(3/2))*sgn(x)/c^(5/2)
```

3.43 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} dx$

Optimal. Leaf size=227

$$-\frac{a^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{bx\left(b^2-12ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{(8ac+b^2+2bc}{}$$

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c*x) + (a*x^2 + b*
x^3 + c*x^4)^(3/2)/(3*x^3) - (a^(3/2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a
+ b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] - (b
*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(16*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.255295, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1921, 1945, 1933, 843, 621, 206, 724}

$$-\frac{a^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{bx\left(b^2-12ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{(8ac+b^2+2bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```


```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c*x) + (a*x^2 + b*
x^3 + c*x^4)^(3/2)/(3*x^3) - (a^(3/2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a
+ b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] - (b
*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(16*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```


Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
```

```
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} + \frac{1}{2}\int \frac{(2a+bx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} + \frac{\int \frac{8a^2c-\frac{1}{2}b(b^2-12ac)x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} + \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{8a^2c-}{x\sqrt{}}}{8c\sqrt{ax^2+bx^3+}}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} + \frac{\left(a^2x\sqrt{a+bx+cx^2}\right)\int \frac{}{x\sqrt{a}}}{\sqrt{ax^2+bx^3+cx}}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} - \frac{\left(2a^2x\sqrt{a+bx+cx^2}\right)\text{Sub}}{\sqrt{ax^2+}}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} - \frac{a^{3/2}x\sqrt{a+bx+cx^2}\tanh^{}}{\sqrt{ax^2+bx^3+}}$$

Mathematica [A] time = 0.23753, size = 166, normalized size = 0.73

$$\frac{x\sqrt{a+x(b+cx)}\left(-48a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\sqrt{a+x(b+cx)}\left(8c\left(4a+cx^2\right)+3b^2+14bcx\right)-3b\left(b^2-12\right.\right.}{48c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(3*b^2 + 14*b*c*x
 + 8*c*(4*a + c*x^2)) - 48*a^(3/2)*c^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])] - 3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])]))/(48*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.005, size = 222, normalized size = 1.

$$-\frac{1}{48\,x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(48\,c^{5/2}a^{3/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)-16\,\left(cx^2+bx+a\right)^{3/2}c^{5/2}-12\,c^{5/2}\sqrt{cx^2+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x)
```

```
[Out] -1/48*(c*x^4+b*x^3+a*x^2)^(3/2)*(48*c^(5/2)*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(
c*x^2+b*x+a)^(1/2))/x)-16*(c*x^2+b*x+a)^(3/2)*c^(5/2)-12*c^(5/2)*(c*x^2+b*x
+a)^(1/2)*x*b-48*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a-6*c^(3/2)*(c*x^2+b*x+a)^(1/2
)*b^2-36*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b*c^2+3*
ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^3*c)/x^3/(c*x^2+b
*x+a)^(3/2)/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4, x)
```

Fricas [A] time = 2.56221, size = 1825, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*s
qrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 3*(b^3 - 12*a*b*c)*s
qrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x
 + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c +
 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(24*a^(3/2)*c^2*x*log
(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*
(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c
*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2
*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))
/(c^2*x), 1/96*(96*sqrt(-a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^3 - 12*a*b*c)*sqrt
(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32
*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(48*sqrt(-a)*a*c^2*x*arc
tan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2
 + a^2*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 +
a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(8*c^3*x^2 + 1
4*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.44 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=219

$$\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}+\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-3\sqrt{a}bx\sqrt{a\cdots}$$

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x) - (a*x^2 + b*x^3 + c*x^
4)^(3/2)/x^4 - (3*Sqrt[a]*b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*
Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*(b^2
+ 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.242014, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1920, 1945, 1933, 843, 621, 206, 724}

$$\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}+\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-3\sqrt{a}bx\sqrt{a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x) - (a*x^2 + b*x^3 + c*x^
4)^(3/2)/x^4 - (3*Sqrt[a]*b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*
Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*(b^2
+ 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
```

```
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^5}\,dx = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4} + \frac{3}{2}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4} + \frac{3\int \frac{4abc+c\left(b^2+4ac\right)x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4} + \frac{\left(3x\sqrt{a+bx+cx^2}\right)\int \frac{4abc+c\left(b^2+4ac\right)x}{x\sqrt{a+bx+cx^2}}}{8c\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4} + \frac{\left(3abx\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,d}{2\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4} - \frac{\left(3abx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x}\right.}{\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4} - \frac{3\sqrt{a}bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a}{2\sqrt{a}\sqrt{}}\right.}{2\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.184463, size = 158, normalized size = 0.72

$$\frac{\sqrt{a+x(b+cx)}\left(3x\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\sqrt{a+x(b+cx)}(x(5b+2cx)-4a)-12\sqrt{a}b\sqrt{c}x\tanh^{-1}\left(\frac{2}{2\sqrt{a}\sqrt{}}\right.\right.}{8\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-4*a + x*(5*b + 2*
c*x)) - 12*Sqrt[a]*b*Sqrt[c]*x*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b
 + c*x)])] + 3*(b^2 + 4*a*c)*x*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)])]))/(8*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.006, size = 254, normalized size = 1.2

$$\frac{1}{8\,ax^4}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(8\,c^{5/2}\left(cx^2+bx+a\right)^{3/2}x^2+12\,c^{5/2}\sqrt{cx^2+bx+a}x^2a-12\,c^{3/2}a^{3/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x)
```

```
[Out] 1/8*(c*x^4+b*x^3+a*x^2)^(3/2)*(8*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^2+12*c^(5/2)
*(c*x^2+b*x+a)^(1/2)*x^2*a-12*c^(3/2)*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+
b*x+a)^(1/2))/x)*x*b-8*(c*x^2+b*x+a)^(5/2)*c^(3/2)+8*c^(3/2)*(c*x^2+b*x+a)^
(3/2)*x*b+18*c^(3/2)*(c*x^2+b*x+a)^(1/2)*x*a*b+12*ln(1/2*(2*(c*x^2+b*x+a)^(
1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*x*a^2*c^2+3*c*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*
c^(1/2)+2*c*x+b)/c^(1/2))*x*a*b^2)/x^4/(c*x^2+b*x+a)^(3/2)/a/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5, x)
```

Fricas [A] time = 2.21845, size = 1735, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4
*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^2 + 4*a*c)*sq
rt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*
x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2
*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8*(6*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(a))/x^3) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a
*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*
x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/16*(24*sqrt(-a)*b*c*
x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 +
a*b*x^2 + a^2*x)) + 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2
 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x)
+ 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8
*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1
/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*
c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**5, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.45 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=219

$$-\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}+\frac{3b\sqrt{cx\cdots}}{\cdots}$$

```
[Out] (-3*(b - 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (a*x^2 + b*x^3 + c*x
^4)^(3/2)/(2*x^5) - (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4]) + (3*b*Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.237785, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1920, 1941, 1933, 843, 621, 206, 724}

$$-\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}+\frac{3b\sqrt{cx\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (-3*(b - 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (a*x^2 + b*x^3 + c*x
^4)^(3/2)/(2*x^5) - (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4]) + (3*b*Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1941

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(A*(m + p*q + (n
 - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
 - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
- q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
```

```
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^6}\,dx = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}+\frac{3}{4}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$$
$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{3}{8}\int \frac{-b^2-4ac-4bcx}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{\left(3x\sqrt{a+bx+cx^2}\right)\int \frac{-b^2-4ac-4bcx}{x\sqrt{a+bx+cx^2}}}{8\sqrt{ax^2+bx^3+cx^4}}$$
$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}+\frac{\left(3bcx\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx}}}{2\sqrt{ax^2+bx^3+cx^4}}$$
$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}+\frac{\left(3bcx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int\right.}{\sqrt{ax^2+bx^3+}}$$
$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}-\frac{3\left(b^2+4ac\right)x\sqrt{a+bx+cx^2}\tan}{8\sqrt{a}\sqrt{ax^2+bx^3+}}$$

Mathematica [A] time = 0.199347, size = 162, normalized size = 0.74

$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(3x^2\left(4ac+b^2\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2\sqrt{a}\left((2a+x(5b-4cx))\sqrt{a+x(b+cx)}-6b\sqrt{c}x^2\tanh\right.\right.}{8\sqrt{a}x^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] -(Sqrt[x^2*(a + x*(b + c*x))]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(2*a + b*x)/(2*S
qrt[a]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[a]*((2*a + x*(5*b - 4*c*x))*Sqrt[a
+ x*(b + c*x)] - 6*b*Sqrt[c]*x^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*
(b + c*x)])])))/(8*Sqrt[a]*x^3*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.006, size = 338, normalized size = 1.5

$$-\frac{1}{8a^2x^5}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(12c^{5/2}a^{5/2}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^2-2c^{5/2}\left(cx^2+bx+a\right)^{3/2}x^3b-4c^{5/2}\left(cx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/8*(c*x^4+b*x^3+a*x^2)^(3/2)*(12*c^(5/2)*a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c
*x^2+b*x+a)^(1/2))/x)*x^2-2*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^3*b-4*c^(5/2)*(c*
x^2+b*x+a)^(3/2)*x^2*a-6*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^3*a*b+3*c^(3/2)*a^(3
/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^2*b^2-12*c^(5/2)*(c*x^2
+b*x+a)^(1/2)*x^2*a^2+2*c^(3/2)*(c*x^2+b*x+a)^(5/2)*x*b-2*c^(3/2)*(c*x^2+b*
x+a)^(3/2)*x^2*b^2+4*(c*x^2+b*x+a)^(5/2)*a*c^(3/2)-6*c^(3/2)*(c*x^2+b*x+a)^
(1/2)*x^2*a*b^2-12*c^2*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/
2))*x^2*a^2*b)/x^5/(c*x^2+b*x+a)^(3/2)/a^2/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6, x)
```

Fricas [A] time = 2.24241, size = 1737, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(12*a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt
(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c
*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1/16*(24*a*b*sqrt(-c)*x^3*arctan(1/2*sqr
t(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))
- 3*(b^2 + 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), 1/8*(6*a*b*sqrt(c)*
x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sq
rt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))
 + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1
/8*(12*a*b*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan
(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 +
a^2*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x
^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**6, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] Timed out
```

3.46 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=257

$$\frac{bx\left(b^2-12ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(-8ac+b^2+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}+\frac{c^{3/2}x\sqrt{a+bx+cx^2}\tanh}{\sqrt{ax^2+bx^3}}$$

```
[Out] ((b^2 - 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a*x^2) - (a*x^2 +
b*x^3 + c*x^4)^(3/2)/(3*x^6) - (b*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(4*a*x^5)
+ (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/(16*a^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (c^(3/2
)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]
```

Rubi [A] time = 0.350564, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1920, 1951, 1941, 1933, 843, 621, 206, 724}

$$\frac{bx\left(b^2-12ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(-8ac+b^2+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}+\frac{c^{3/2}x\sqrt{a+bx+cx^2}\tanh}{\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] ((b^2 - 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a*x^2) - (a*x^2 +
b*x^3 + c*x^4)^(3/2)/(3*x^6) - (b*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(4*a*x^5)
+ (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/(16*a^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (c^(3/2
)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 1941

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(A*(m + p*q + (n
 - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
 - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
- q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^7}\,dx = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} + \frac{1}{2}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx$$

$$= -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5} - \frac{\int \frac{\left(\frac{1}{2}\left(b^2-8ac\right)-bcx\right)\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx}{4a}$$

$$= \frac{\left(b^2-8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5} + \int$$

$$= \frac{\left(b^2-8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5} + \left(x\sqrt{\right.$$

$$= \frac{\left(b^2-8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5} + \left(c^2x\right.$$

$$= \frac{\left(b^2-8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5} + \left(2c^2\right.$$

$$= \frac{\left(b^2-8ac+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6} - \frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5} + b\left(b^2\right.$$

Mathematica [A] time = 0.306707, size = 175, normalized size = 0.68

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(3bx^3\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}\left(\sqrt{a+x(b+cx)}\left(8a^2+2ax(7b+16cx)+3b^2x^2\right)-24\right.\right.}{48a^{3/2}x^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(3*b*(b^2 - 12*a*c)*x^3*ArcTanh[(2*a + b*x)/(2
*Sqrt[a]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[a]*(Sqrt[a + x*(b + c*x)]*(8*a^2
+ 3*b^2*x^2 + 2*a*x*(7*b + 16*c*x)) - 24*a*c^(3/2)*x^3*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/(48*a^(3/2)*x^4*Sqrt[a + x*(b + c*x)]
)
```

Maple [A] time = 0.007, size = 435, normalized size = 1.7

$$\frac{1}{48\,x^6a^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(32\,c^{7/2}\left(cx^2+bx+a\right)^{3/2}x^4a-36\,c^{5/2}a^{5/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^3b+48\,c^{7/2}\sqrt{c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(3/2)*(32*c^(7/2)*(c*x^2+b*x+a)^(3/2)*x^4*a-36*c^(
5/2)*a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^3*b+48*c^(7/2)
*(c*x^2+b*x+a)^(1/2)*x^4*a^2-2*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^4*b^2-32*c^(5/
2)*(c*x^2+b*x+a)^(5/2)*x^2*a+28*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^3*a*b-6*c^(5/
2)*(c*x^2+b*x+a)^(1/2)*x^4*a*b^2+3*c^(3/2)*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c
*x^2+b*x+a)^(1/2))/x)*x^3*b^3+60*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^3*a^2*b+2*c^
```

```
(3/2)*(c*x^2+b*x+a)^(5/2)*x^2*b^2-2*c^(3/2)*(c*x^2+b*x+a)^(3/2)*x^3*b^3+4*c
^(3/2)*(c*x^2+b*x+a)^(5/2)*x*a*b-6*c^(3/2)*(c*x^2+b*x+a)^(1/2)*x^3*a*b^3-16
*(c*x^2+b*x+a)^(5/2)*a^2*c^(3/2)+48*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2
*c*x+b)/c^(1/2))*x^3*a^3*c^3)/x^6/(c*x^2+b*x+a)^(3/2)/a^3/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7, x)
```

Fricas [A] time = 2.49978, size = 1871, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)*s
qrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b
*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14
*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/96*(96*a^2*sqrt
(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2
*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 +
 (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2
+ 32*a^2*c)*x^2))/(a^2*x^4), 1/48*(24*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b
*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*
x)/x) - 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/4
8*(48*a^2*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*ar
ctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^
2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2
+ 32*a^2*c)*x^2))/(a^2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2 \left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**7, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.47 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=197

$$\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}-\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}-\frac{\left(ax^2+b\right.}{}$$

```
[Out] -((b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a*x^3) + (b*(3*b^2 - 20*a
*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^2*x^2) - ((b + 6*c*x)*Sqrt[a*x^2 + b
*x^3 + c*x^4])/(8*x^4) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(4*x^7) - (3*(b^2 -
4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/
(128*a^(5/2))
```

Rubi [A] time = 0.362848, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1920, 1941, 1951, 12, 1904, 206}

$$\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}-\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}-\frac{\left(ax^2+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -((b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a*x^3) + (b*(3*b^2 - 20*a
*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^2*x^2) - ((b + 6*c*x)*Sqrt[a*x^2 + b
*x^3 + c*x^4])/(8*x^4) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(4*x^7) - (3*(b^2 -
4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/
(128*a^(5/2))
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1941

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(A*(m + p*q + (n
 - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
 - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
- q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^8}\,dx = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}+\frac{3}{8}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$$

$$= -\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}+\frac{1}{16}\int \frac{b^2-12ac-4bcx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}-\int \frac{\frac{1}{2}}{}$$

$$= -\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b+6cx)\sqrt{ax^2+bx^3}}{8x^4}$$

$$= -\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b+6cx)\sqrt{ax^2+bx^3}}{8x^4}$$

$$= -\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b+6cx)\sqrt{ax^2+bx^3}}{8x^4}$$

$$= -\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b+6cx)\sqrt{ax^2+bx^3}}{8x^4}$$

Mathematica [A] time = 0.116674, size = 141, normalized size = 0.72

$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(2\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}\left(8a^2+4ax(2b+5cx)-3b^2x^2\right)+3x^4\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx}}\right.\right.}{128a^{5/2}x^5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -(Sqrt[x^2*(a + x*(b + c*x))]*(2*Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)]*
(8*a^2 - 3*b^2*x^2 + 4*a*x*(2*b + 5*c*x)) + 3*(b^2 - 4*a*c)^2*x^4*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(128*a^(5/2)*x^5*Sqrt[a + x
*(b + c*x)])
```

Maple [B] time = 0.009, size = 501, normalized size = 2.5

$$-\frac{1}{128\,x^7a^4}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(48\,c^2a^{7/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^4+24\,c^2\left(cx^2+bx+a\right)^{3/2}x^5ab-24\,ca^{5/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x)
```

```
[Out] -1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(48*c^2*a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*x
^2+b*x+a)^(1/2))/x)*x^4+24*c^2*(c*x^2+b*x+a)^(3/2)*x^5*a*b-24*c*a^(5/2)*ln(
(2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^4*b^2-16*c^2*(c*x^2+b*x+a)^(3/
2)*x^4*a^2+24*c^2*(c*x^2+b*x+a)^(1/2)*x^5*a^2*b-2*c*(c*x^2+b*x+a)^(3/2)*x^5
*b^3-48*c^2*(c*x^2+b*x+a)^(1/2)*x^4*a^3-24*c*(c*x^2+b*x+a)^(5/2)*x^3*a*b+20
*c*(c*x^2+b*x+a)^(3/2)*x^4*a*b^2-6*c*(c*x^2+b*x+a)^(1/2)*x^5*a*b^3+3*a^(3/2
)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^4*b^4+16*c*(c*x^2+b*x+a)^
(5/2)*x^2*a^2+36*c*(c*x^2+b*x+a)^(1/2)*x^4*a^2*b^2+2*(c*x^2+b*x+a)^(5/2)*x^
3*b^3-2*(c*x^2+b*x+a)^(3/2)*x^4*b^4+4*(c*x^2+b*x+a)^(5/2)*x^2*a*b^2-6*(c*x^
2+b*x+a)^(1/2)*x^4*a*b^4-16*(c*x^2+b*x+a)^(5/2)*x*a^2*b+32*(c*x^2+b*x+a)^(5
/2)*a^3)/x^7/(c*x^2+b*x+a)^(3/2)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8, x)
```

Fricas [A] time = 2.3415, size = 743, normalized size = 3.77

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{a}x^5\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\left(24\,a^3bx+16\,a^4-\left(3\,ab^3-20\,a^2\right.\right.}{256\,a^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (b^2
 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a)
)/x^3) - 4*(24*a^3*b*x + 16*a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 +
 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^3*x^5), 1/128*(3*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b
*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(24*a^3*b*x + 16*a^4 -
(3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3
 + a*x^2))/(a^3*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**8, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] Timed out
```

3.48 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=249

$$-\frac{(128a^2c^2-100ab^2c+15b^4)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2}+\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}+\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{x(2a-}{2\sqrt{a}\sqrt{ax^2-}}\right.}{256a^{7/2}}$$

```
[Out] -((b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(80*a*x^4) + (b*(5*b^2 - 28*a*
c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(320*a^2*x^3) - ((15*b^4 - 100*a*b^2*c + 12
8*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(640*a^3*x^2) - (3*(b + 4*c*x)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(40*x^5) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(5*x^8) +
 (3*b*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3
 + c*x^4])])/(256*a^(7/2))
```

Rubi [A] time = 0.503853, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1920, 1941, 1951, 12, 1904, 206}

$$-\frac{(128a^2c^2-100ab^2c+15b^4)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2}+\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}+\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{x(2a-}{2\sqrt{a}\sqrt{ax^2-}}\right.}{256a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -((b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(80*a*x^4) + (b*(5*b^2 - 28*a*
c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(320*a^2*x^3) - ((15*b^4 - 100*a*b^2*c + 12
8*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(640*a^3*x^2) - (3*(b + 4*c*x)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(40*x^5) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(5*x^8) +
 (3*b*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3
 + c*x^4])])/(256*a^(7/2))
```

Rule 1920

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q
 + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1941

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(A*(m + p*q + (n
 - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
 - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
- q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
```

```
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^9}\,dx = -\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}+\frac{3}{10}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$$

$$= -\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}+\frac{3}{160}\int \frac{2\left(b^2-8ac\right)-4bcx}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}-\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}-\int \frac{b(\cdots}{\cdots}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{3(b+4cx)\sqrt{ax^2+bx^3\cdots}}{40x^5}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{\left(15b^4-100ab^2c+128\cdots\right.}{640\cdots}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{\left(15b^4-100ab^2c+128\cdots\right.}{640\cdots}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{\left(15b^4-100ab^2c+128\cdots\right.}{640\cdots}$$

$$= -\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{\left(15b^4-100ab^2c+128\cdots\right.}{640\cdots}$$

Mathematica [A] time = 0.176466, size = 177, normalized size = 0.71

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(15bx^5\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}\sqrt{a+x(b+cx)}\left(8a^2x^2\left(b^2+7bcx+16c^2x^2\right)+16a\right.}{1280a^{7/2}x^6\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(128*a^4 + 1
5*b^4*x^4 - 10*a*b^2*x^3*(b + 10*c*x) + 16*a^3*x*(11*b + 16*c*x) + 8*a^2*x^
2*(b^2 + 7*b*c*x + 16*c^2*x^2)) + 15*b*(b^2 - 4*a*c)^2*x^5*ArcTanh[(2*a + b
*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(1280*a^(7/2)*x^6*Sqrt[a + x*(b +
c*x)])
```

Maple [B] time = 0.008, size = 534, normalized size = 2.1

$$-\frac{1}{1280\,x^8a^5}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(-240\,c^2a^{7/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^5b-120\,\left(cx^2+bx+a\right)^{3/2}c^2x^6ab^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x)
```

```
[Out] -1/1280*(c*x^4+b*x^3+a*x^2)^(3/2)*(-240*c^2*a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(
c*x^2+b*x+a)^(1/2))/x)*x^5*b-120*(c*x^2+b*x+a)^(3/2)*c^2*x^6*a*b^2+120*c*a^
(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^5*b^3+80*(c*x^2+b*x+a
)^(3/2)*c^2*x^5*a^2*b+10*(c*x^2+b*x+a)^(3/2)*c*x^6*b^4-120*(c*x^2+b*x+a)^(1
/2)*c^2*x^6*a^2*b^2+120*(c*x^2+b*x+a)^(5/2)*c*x^4*a*b^2-100*(c*x^2+b*x+a)^(
3/2)*c*x^5*a*b^3+240*(c*x^2+b*x+a)^(1/2)*c^2*x^5*a^3*b+30*(c*x^2+b*x+a)^(1/
2)*c*x^6*a*b^4-15*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^5
*b^5-80*(c*x^2+b*x+a)^(5/2)*c*x^3*a^2*b-10*(c*x^2+b*x+a)^(5/2)*x^4*b^4+10*(
c*x^2+b*x+a)^(3/2)*x^5*b^5-180*(c*x^2+b*x+a)^(1/2)*c*x^5*a^2*b^3-20*(c*x^2+
b*x+a)^(5/2)*x^3*a*b^3+30*(c*x^2+b*x+a)^(1/2)*x^5*a*b^5+80*(c*x^2+b*x+a)^(5
/2)*x^2*a^2*b^2-160*(c*x^2+b*x+a)^(5/2)*x*a^3*b+256*(c*x^2+b*x+a)^(5/2)*a^4
)/x^8/(c*x^2+b*x+a)^(3/2)/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9, x)
```

Fricas [A] time = 2.71046, size = 898, normalized size = 3.61

$$\left[\frac{15\left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)\sqrt{a}x^6 \log\left(-\frac{8\,abx^2 + \left(b^2 + 4\,ac\right)x^3 + 8\,a^2x + 4\,\sqrt{cx^4 + bx^3 + ax^2}(bx + 2\,a)\sqrt{a}}{x^3}\right) - 4\left(176\,a^4bx + 128\,a^5 + \left(15\,ab^4 - 10\right.\right.}{2560\,a^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^6*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(a))/x^3) - 4*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*b^2*c + 128*a^3
*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2)*sq
rt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6), -1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2
*b*c^2)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(176*a^4*b*x + 128*a^5 + (15*a*b^4 -
 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3
*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**9,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] Timed out
```

3.49 $\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=143

$$\frac{x\left(3b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}$$

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^
2*x) + ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.174007, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1928, 1949, 12, 1914, 621, 206}

$$\frac{x\left(3b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^
2*x) + ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1928

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - 2*n + q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(
p + 1))/(c*(m + p*q + 2*(n - q)*p + 1)), x] - Dist[1/(c*(m + p*q + 2*(n - q
)*p + 1)), Int[x^(m - 2*(n - q))*(a*(m + p*q - 2*(n - q) + 1) + b*(m + p*q
+ (n - q)*(p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
&& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && Rational
Q[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx &= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(a+\frac{3bx}{2}\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\int \frac{\left(3b^2-4ac\right)x}{4\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c^2}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\left(3b^2-4ac\right)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c^2}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\left(\left(3b^2-4ac\right)x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c^2\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\left(\left(3b^2-4ac\right)x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x\right.}{4c^2\sqrt{ax^2+bx^3+cx^4}}\\
&= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\left(3b^2-4ac\right)x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}
$$

Mathematica [A] time = 0.136673, size = 105, normalized size = 0.73

$$
\frac{x\left(\left(3b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}(2cx-3b)(a+x(b+cx))\right)}{8c^{5/2}\sqrt{x^2(a+x(b+cx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*(2*Sqrt[c]*(-3*b + 2*c*x)*(a + x*(b + c*x)) + (3*b^2 - 4*a*c)*Sqrt[a + x
*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(8*c^(
5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.006, size = 144, normalized size = 1.

$$\frac{x}{8}\sqrt{cx^2+bx+a}\left(4\sqrt{cx^2+bx+a}c^{5/2}x-6\sqrt{cx^2+bx+a}c^{3/2}b-4\ln\left(1/2\,\frac{2\sqrt{cx^2+bx+a}\sqrt{c}+2\,cx+b}{\sqrt{c}}\right)ac^2+3\ln\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/8*x*(c*x^2+b*x+a)^(1/2)*(4*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x-6*(c*x^2+b*x+a)^
(1/2)*c^(3/2)*b-4*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a
*c^2+3*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^2*c)/(c*x^
4+b*x^3+a*x^2)^(1/2)/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Fricas [A] time = 1.56927, size = 514, normalized size = 3.59

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}\left(2\,c^2x-3\,bc\right)}{16\,c^3x},-\frac{\left(3\,b^2-4\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^
4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4
 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x), -1/8*((3*b^2 - 4*a*c)*sqrt(-c
)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 +
b*c*x^2 + a*c*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(1/2),x)

[Out] Integral(x**3/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^4 + bx^3 + ax^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

3.50 $\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])
```

Rubi [A] time = 0.0776804, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1917, 1914, 621, 206}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])
```

Rule 1917

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n)*(a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
)/(2*c*(p + 1)), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*
x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p,
 q] && EqQ[m + p*(n - 1) - 1, 0]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{b\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{\left(bx\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2c\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{\left(bx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.0540195, size = 89, normalized size = 0.86

$$\frac{x\left(2\sqrt{c}(a+x(b+cx)) - b\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*(2*Sqrt[c]*(a + x*(b + c*x)) - b*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x
))])
```

Maple [A] time = 0.006, size = 88, normalized size = 0.9

$$\frac{x}{2}\sqrt{cx^2+bx+a}\left(2\sqrt{cx^2+bx+a}c^{3/2} - b\ln\left(\frac{1}{2}\left(2\sqrt{cx^2+bx+a}\sqrt{c}+2cx+b\right)\frac{1}{\sqrt{c}}\right)c\right)\frac{1}{\sqrt{cx^4+bx^3+ax^2}}c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*c^(3/2)-b*ln(1/2*(2*(c*x^2
+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*c)/(c*x^4+b*x^3+a*x^2)^(1/2)/c^(5/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Fricas [A] time = 1.59089, size = 429, normalized size = 4.17

$$\left[\frac{b\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+4\,\sqrt{cx^4+bx^3+ax^2}c}{4\,c^2x}, \frac{b\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\left(c^2x^3+bcx^2+acx\right)}\right)}{2\,c^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2
)*c)/(c^2*x), 1/2*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^
2)*c)/(c^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [A] time = 1.15572, size = 146, normalized size = 1.42

$$\frac{b\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{b\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)-2\,\sqrt{a}c}{\left(\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)^2-c\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c) + (b*(s
qrt(c + b/x + a/x^2) - sqrt(a)/x) - 2*sqrt(a)*c)/(((sqrt(c + b/x + a/x^2) -
 sqrt(a)/x)^2 - c)*c)
```

3.51 $\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}} \, dx$

Optimal. Leaf size=71

$$\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.0366343, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1914, 621, 206}

$$\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}}$$
$$= \frac{\left(2x\sqrt{a+bx+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$
$$= \frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.0412742, size = 66, normalized size = 0.93

$$\frac{x\sqrt{a+bx+cx^2}\log\left(2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x + c*x^2]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/
(Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.006, size = 65, normalized size = 0.9

$$x\sqrt{cx^2+bx+a}\ln\left(\frac{1}{2}\left(2\sqrt{cx^2+bx+a}\sqrt{c}+2\,cx+b\right)\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{cx^4+bx^3+ax^2}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*(c*x^2+b*x+a)^(
1/2)*c^(1/2)+2*c*x+b)/c^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Fricas [A] time = 1.54597, size = 298, normalized size = 4.2

$$\left[\frac{\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\left(c^2x^3+bcx^2+acx\right)}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b
)*sqrt(c) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -sqrt(-c)*arctan(1/2*sqrt(c*x^4 +
b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [A] time = 1.15887, size = 50, normalized size = 0.7

$$-\frac{2\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/sqrt(-c)
```

3.52 $\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])
```

Rubi [A] time = 0.0158112, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0211026, size = 70, normalized size = 1.56

$$-\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -((x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*
x^2])])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))]))
```

Maple [A] time = 0.005, size = 66, normalized size = 1.5

$$-x\sqrt{cx^2+bx+a}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{cx^4+bx^3+ax^2}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2*a^
(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Fricas [A] time = 1.61596, size = 300, normalized size = 6.67

$$\left[\frac{\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\left(acx^3+abx^2+a^2x\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

Giac [A] time = 1.15699, size = 80, normalized size = 1.78

$$-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+\frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

3.53 $\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}} \, dx$

Optimal. Leaf size=77

$$\frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt
[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))
```

Rubi [A] time = 0.0540936, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1927, 1904, 206}

$$\frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt
[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))
```

Rule 1927

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
))/(2*a*(n - q)*(p + 1)), x] - Dist[b/(2*a), Int[x^(m + n - q)*(a*x^q + b*x
^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p,
 -1] && LtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, -2*(n - q)*(p + 1)
]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} - \frac{b\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a}$$
$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{a}$$
$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} + \frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.0555978, size = 89, normalized size = 1.16

$$\frac{bx\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a}(a+x(b+cx))}{2a^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[a]*(a + x*(b + c*x)) + b*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*
x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(2*a^(3/2)*Sqrt[x^2*(a + x*(b + c*x)
)])
```

Maple [A] time = 0.006, size = 88, normalized size = 1.1

$$-\frac{1}{2}\sqrt{cx^2+bx+a}\left(2\sqrt{cx^2+bx+a}a^{3/2} - b\ln\left(\frac{1}{x}\left(2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)ax\right)\frac{1}{\sqrt{cx^4+bx^3+ax^2}}a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/2*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*a^(3/2)-b*ln((2*a+b*x+2*a^(
1/2)*(c*x^2+b*x+a)^(1/2))/x)*a*x)/(c*x^4+b*x^3+a*x^2)^(1/2)/a^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x), x)
```

Fricas [A] time = 1.73191, size = 444, normalized size = 5.77

$$\left[\frac{\sqrt{a}bx^2\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,\sqrt{cx^4+bx^3+ax^2}a}{4\,a^2x^2},-\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{\cdots}}{2\left(acx^3+abx^2+a^2x\right)}\right.}{2\,a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(
c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a
*x^2)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*a)/(a^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x + c*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.54 $\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=119

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(4*a^2*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^
2 + b*x^3 + c*x^4])])/(8*a^(5/2))
```

Rubi [A] time = 0.148656, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1929, 1951, 12, 1904, 206}

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(4*a^2*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^
2 + b*x^3 + c*x^4])])/(8*a^(5/2))
```

Rule 1929

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
)/(a*(m + p*q + 1)), x] - Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*(b*(m
 + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ
[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{\int \frac{-\frac{3b}{2}-cx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\int \frac{-\frac{3b^2}{4}+ac}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a^2}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}+\frac{\left(3b^2-4ac\right)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8a^2}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\left(3b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{4a^2}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0955229, size = 112, normalized size = 0.94

$$\frac{x^2\left(-\left(3b^2-4ac\right)\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}(2a-3bx)(a+x(b+cx))}{8a^{5/2}x\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[a]*(2*a - 3*b*x)*(a + x*(b + c*x)) - (3*b^2 - 4*a*c)*x^2*Sqrt[a +
x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(8*a^(
5/2)*x*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.006, size = 152, normalized size = 1.3

$$-\frac{1}{8x}\sqrt{cx^2+bx+a}\left(4\sqrt{cx^2+bx+a}a^{5/2}-6a^{3/2}\sqrt{cx^2+bx+a}xb-4c\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)a^2x^2+3\ln\left(\frac{2}{\ }\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b*x+a)^(1/2)*(4*(c*x^2+b*x+a)^(1/2)*a^(5/2)-6*a^(3/2)*(c*x^2+b*
x+a)^(1/2)*x*b-4*c*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*a^2*x^2+3*
```

```
ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^2*a*b^2)/x/(c*x^4+b*x^3+a*x
^2)^(1/2)/a^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2), x)
```

Fricas [A] time = 1.7908, size = 527, normalized size = 4.43

$$\left[-\frac{\left(3\,b^2 - 4\,ac\right)\sqrt{a}x^3 \log\left(-\frac{8\,abx^2 + \left(b^2 + 4\,ac\right)x^3 + 8\,a^2x + 4\,\sqrt{cx^4 + bx^3 + ax^2}(bx + 2\,a)\sqrt{a}}{x^3}\right) - 4\,\sqrt{cx^4 + bx^3 + ax^2}\left(3\,abx - 2\,a^2\right)}{16\,a^3x^3}, \frac{\left(3\,b^2 - 4\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8
*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/8*((3*b^2 - 4*a*c)*sq
rt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c
*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))
/(a^3*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^2\left(a + bx + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x + c*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.55 $\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c^3x\left(b^2-4ac\right)}+\frac{3x\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx}}\right.}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + ((5*b^2 -
 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c^2*(b^2 - 4*a*c)) - (b*(15*b^2 -
52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^3*(b^2 - 4*a*c)*x) - (2*b*x*Sqrt[
a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + (3*(5*b^2 - 4*a*c)*x*Sqrt[a + b
*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7
/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.50583, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c^3x\left(b^2-4ac\right)}+\frac{3x\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx}}\right.}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + ((5*b^2 -
 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c^2*(b^2 - 4*a*c)) - (b*(15*b^2 -
52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^3*(b^2 - 4*a*c)*x) - (2*b*x*Sqrt[
a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + (3*(5*b^2 - 4*a*c)*x*Sqrt[a + b
*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7
/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1923

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x
^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(
(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n -
q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c
*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
 -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
```

```
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{x^3(6a+3bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{2\int \frac{x^2\left(6ab+\frac{3}{2}\left(5b^2-12ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{3c\left(b^2-4ac\right)}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} -$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+}}{4c^3\left(b^2-4ac\right)}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+}}{4c^3\left(b^2-4ac\right)}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+}}{4c^3\left(b^2-4ac\right)}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+}}{4c^3\left(b^2-4ac\right)}$$

$$= \frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)} - \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+}}{4c^3\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.258017, size = 183, normalized size = 0.7

$$\frac{x\left(2\sqrt{c}\left(4a^2c(6cx-13b)+a\left(-62b^2cx+15b^3-20bc^2x^2+8c^3x^3\right)+b^2x\left(15b^2+5bcx-2c^2x^2\right)\right)-3\left(16a^2c^2-24ab^2c+\right.}{8c^{7/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x) + b^2*x*(15*b^2 + 5*b*c*x - 2*c^2*x^
2) + a*(15*b^3 - 62*b^2*c*x - 20*b*c^2*x^2 + 8*c^3*x^3)) - 3*(5*b^4 - 24*a*
b^2*c + 16*a^2*c^2)*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])]))/(8*c^(7/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x)
)])
```

Maple [A] time = 0.009, size = 283, normalized size = 1.1

$$-\frac{x^3\left(cx^2+bx+a\right)}{32\,ac-8\,b^2}\left(-16\,c^{9/2}x^3a+4\,c^{7/2}x^3b^2+40\,c^{7/2}x^2ab-10\,c^{5/2}x^2b^3-48\,c^{7/2}xa^2+124\,c^{5/2}xab^2-30\,c^{3/2}xb^4+48\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/8*x^3*(c*x^2+b*x+a)/c^(9/2)*(-16*c^(9/2)*x^3*a+4*c^(7/2)*x^3*b^2+40*c^(7
/2)*x^2*a*b-10*c^(5/2)*x^2*b^3-48*c^(7/2)*x*a^2+124*c^(5/2)*x*a*b^2-30*c^(3
/2)*x*b^4+48*(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*
x+b)/c^(1/2))*a^2*c^3-72*(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*
c^(1/2)+2*c*x+b)/c^(1/2))*a*b^2*c^2+15*(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*(c*x^2
+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^4*c+104*c^(5/2)*a^2*b-30*c^(3/2)*
a*b^3)/(c*x^4+b*x^3+a*x^2)^(3/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A] time = 2.27583, size = 1316, normalized size = 5.02

$$\left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^3+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^2+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x\right)\sqrt{c}\log\left(-\frac{8\,c^2x^3+8\,bcx^2-}{}\right.}{16\left(\left(b^2c^5-4\,ac\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(c)*log(
-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c)
 + (b^2 + 4*a*c)*x)/x) + 4*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^
4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^
3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*
a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x), -1/8*(3*((5*b^4*c - 24*a*b^2*c^2
 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 2
4*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(15*a*b^3*c - 52*a^
2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4
*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 -
 4*a*c^6)*x^3 + (b^3*c^4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**7/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

3.56 $\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=201

$$\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2x\left(b^2-4ac\right)}+\frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)}-\frac{3bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+\ldots}{2\sqrt{c}\sqrt{a\ldots}}\right.}{2c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 +
b*x^3 + c*x^4])/(c^2*(b^2 - 4*a*c)*x) - (3*b*x*Sqrt[a + b*x + c*x^2]*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*Sqrt[a*x^2 + b
*x^3 + c*x^4])
```

Rubi [A] time = 0.304659, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2x\left(b^2-4ac\right)}+\frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)}-\frac{3bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+\ldots}{2\sqrt{c}\sqrt{a\ldots}}\right.}{2c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 +
b*x^3 + c*x^4])/(c^2*(b^2 - 4*a*c)*x) - (3*b*x*Sqrt[a + b*x + c*x^2]*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*Sqrt[a*x^2 + b
*x^3 + c*x^4])
```

Rule 1923

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x
^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(
(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n -
q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c
*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
 -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
```

```
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{x^2(4a+2bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac}$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\int \frac{x\left(2ab+\left(3b^2-8ac\right)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{c\left(b^2-4ac\right)}$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2\left(b^2-4ac\right)x} -$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2\left(b^2-4ac\right)x} -$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2\left(b^2-4ac\right)x} -$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2\left(b^2-4ac\right)x} -$$

$$= \frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)} + \frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2\left(b^2-4ac\right)x} -$$

Mathematica [A] time = 0.15881, size = 141, normalized size = 0.7

$$\frac{x\left(2\sqrt{c}\left(8a^2c+a\left(-3b^2+10bcx+4c^2x^2\right)-b^2x(3b+cx)\right)+3b\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2c^{5/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(8*a^2*c - b^2*x*(3*b + c*x) + a*(-3*b^2 + 10*b*c*x + 4*c^2*x
^2)) + 3*b*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b +
 c*x))])
```

Maple [A] time = 0.006, size = 199, normalized size = 1.

$$\frac{x^3\left(cx^2+bx+a\right)}{8\,ac-2\,b^2}\left(8\,c^{7/2}x^2a-2\,c^{5/2}x^2b^2+20\,c^{5/2}xab-6\,c^{3/2}xb^3+16\,c^{5/2}a^2-6\,c^{3/2}ab^2-12\,\ln\left(1/2\,\frac{2\,\sqrt{cx^2+bx+a}\sqrt{c}+}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/2*x^3*(c*x^2+b*x+a)/c^(7/2)*(8*c^(7/2)*x^2*a-2*c^(5/2)*x^2*b^2+20*c^(5/2)
*x*a*b-6*c^(3/2)*x*b^3+16*c^(5/2)*a^2-6*c^(3/2)*a*b^2-12*ln(1/2*(2*(c*x^2+b
*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*(c*x^2+b*x+a)^(1/2)*a*b*c^2+3*ln(1/2*
(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*(c*x^2+b*x+a)^(1/2)*b^3*c)
/(c*x^4+b*x^3+a*x^2)^(3/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^6}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A] time = 2.09024, size = 1026, normalized size = 5.1

$$\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^3+\left(b^4-4\,ab^2c\right)x^2+\left(ab^3-4\,a^2bc\right)x\right)\sqrt{c}\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+4\,\sqrt{cx^4+}}{4\left(\left(b^2c^4-4\,ac^5\right)x^3+\left(b^3c^3-4\,abc^4\right)x^2+\left(ab^2c^3-4\,a^2c^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*b
*c)*x)*sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*
(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(
3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x)
)/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2
*c^4)*x), 1/2*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3
- 4*a^2*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a
*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x))/(
(b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2*c^
4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A] time = 1.16984, size = 263, normalized size = 1.31

$$\frac{2\left(\frac{b^3c^2-3\,abc^3}{b^2c^4-4\,ac^5}+\frac{ab^2c^2-2\,a^2c^3}{\left(b^2c^4-4\,ac^5\right)x}\right)}{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}}+\frac{3\,b\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}+\frac{b\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)-2\sqrt{a}c}{\left(\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)^2-c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*((b^3*c^2 - 3*a*b*c^3)/(b^2*c^4 - 4*a*c^5) + (a*b^2*c^2 - 2*a^2*c^3)/((b^
2*c^4 - 4*a*c^5)*x))/sqrt(c + b/x + a/x^2) + 3*b*arctan((sqrt(c + b/x + a/x
^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c^2) + (b*(sqrt(c + b/x + a/x^2) - sqr
t(a)/x) - 2*sqrt(a)*c)/(((sqrt(c + b/x + a/x^2) - sqrt(a)/x)^2 - c)*c^2)
```

3.57 $\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}} dx$

Optimal. Leaf size=153

$$\frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx\left(b^2-4ac\right)}+\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)*x) + (x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*Sqrt[a*x^2 +
b*x^3 + c*x^4])
```

Rubi [A] time = 0.175293, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx\left(b^2-4ac\right)}+\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)*x) + (x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*Sqrt[a*x^2 +
b*x^3 + c*x^4])
```

Rule 1923

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x
^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(
(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n -
q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c
*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
 -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac}$$

$$= \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x}+\frac{2\int \frac{\left(b^2-4ac\right)x}{2\sqrt{ax^2+bx^3+cx^4}}\,dx}{c\left(b^2-4ac\right)}$$

$$= \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x}+\frac{\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{c}$$

$$= \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x}+\frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x}+\frac{\left(2x\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x}\right.}{c\sqrt{ax^2+bx^3+c}}$$

$$= \frac{2x^2(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)x}+\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx}}\right.}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.122982, size = 112, normalized size = 0.73

$$-\frac{x\left(2\sqrt{c}\left(-ab+2acx+b^2(-x)\right)+\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{3/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(2*Sqrt[c]*(-(a*b) - b^2*x + 2*a*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b +
c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(c^(3/2)*(-b
^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))]))
```

Maple [A] time = 0.006, size = 166, normalized size = 1.1

$$-\frac{x^3\left(cx^2+bx+a\right)}{4\,ac-b^2}\left(4\,c^{5/2}xa-2\,c^{3/2}xb^2-2\,c^{3/2}ab-4\,\ln\left(1/2\,\frac{2\,\sqrt{cx^2+bx+a}\sqrt{c}+2\,cx+b}{\sqrt{c}}\right)\sqrt{cx^2+bx+a}ac^2+\ln\left(\frac{1}{2}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -x^3*(c*x^2+b*x+a)/c^(5/2)*(4*c^(5/2)*x*a-2*c^(3/2)*x*b^2-2*c^(3/2)*a*b-4*l
n(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*(c*x^2+b*x+a)^(1/2)*
a*c^2+ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*(c*x^2+b*x+a)
^(1/2)*b^2*c)/(c*x^4+b*x^3+a*x^2)^(3/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A] time = 2.15883, size = 873, normalized size = 5.71

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^3+\left(b^3-4\,abc\right)x^2+\left(ab^2-4\,a^2c\right)x\right)\sqrt{c}\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)-4\,\sqrt{cx^4+bx^3}}{2\left(\left(b^2c^3-4\,ac^4\right)x^3+\left(b^3c^2-4\,abc^3\right)x^2+\left(ab^2c^2-4\,a^2c^3\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*s
qrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
 b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c +
(b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*x^2
+ (a*b^2*c^2 - 4*a^2*c^3)*x), -(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^
2 + (a*b^2 - 4*a^2*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(a*b*c + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4
```

```
*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A] time = 1.19216, size = 149, normalized size = 0.97

$$-\frac{2\left(\frac{abc}{\left(b^2c^2-4\,ac^3\right)x}+\frac{b^2c-2\,ac^2}{b^2c^2-4\,ac^3}\right)}{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}}-\frac{2\,\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(a*b*c/((b^2*c^2 - 4*a*c^3)*x) + (b^2*c - 2*a*c^2)/(b^2*c^2 - 4*a*c^3))/
sqrt(c + b/x + a/x^2) - 2*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-
c))/(sqrt(-c)*c)
```

3.58 $\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.0399566, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1916}

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1916

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[(x^((n - 1)/2)*(4*a + 2*b*x))/((b^2 - 4*a*c)*Sqrt[a*x
^(n - 1) + b*x^n + c*x^(n + 1)]), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (3
*n - 1)/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.0741147, size = 37, normalized size = 0.92

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.004, size = 53, normalized size = 1.3

$$-2\,\frac{(cx^2+bx+a)(bx+2\,a)\,x^3}{(4\,ac-b^2)\,(cx^4+bx^3+ax^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2*(c*x^2+b*x+a)*(b*x+2*a)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A] time = 1.98544, size = 150, normalized size = 3.75

$$\frac{2\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)}{\left(b^2c-4\,ac^2\right)x^3+\left(b^3-4\,abc\right)x^2+\left(ab^2-4\,a^2c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)/((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4
*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A] time = 1.19208, size = 61, normalized size = 1.52

$$\frac{2\left(\frac{b}{b^2-4\,ac}+\frac{2\,a}{\left(b^2-4\,ac\right)x}\right)}{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] 2*(b/(b^2 - 4*a*c) + 2*a/((b^2 - 4*a*c)*x))/sqrt(c + b/x + a/x^2)
```

3.59 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=39

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.0402122, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1915}

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1915

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[(-2*x^((n - 1)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*
x^(n - 1) + b*x^n + c*x^(n + 1)]), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (
3*(n - 1))/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.0244011, size = 36, normalized size = 0.92

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.003, size = 52, normalized size = 1.3

$$2\,\frac{(cx^2+bx+a)(2\,cx+b)\,x^3}{(4\,ac-b^2)(cx^4+bx^3+ax^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 2*(c*x^2+b*x+a)*(2*c*x+b)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A] time = 1.94596, size = 151, normalized size = 3.87

$$-\frac{2\sqrt{cx^4 + bx^3 + ax^2}(2\,cx + b)}{\left(b^2c - 4\,ac^2\right)x^3 + \left(b^3 - 4\,abc\right)x^2 + \left(ab^2 - 4\,a^2c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)/((b^2*c - 4*a*c^2)*x^3 + (b^3 -
4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A] time = 1.16545, size = 61, normalized size = 1.56

$$-\frac{2\left(\frac{2\,c}{b^2-4\,ac} + \frac{b}{\left(b^2-4\,ac\right)x}\right)}{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] -2*(2*c/(b^2 - 4*a*c) + b/((b^2 - 4*a*c)*x))/sqrt(c + b/x + a/x^2)
```

3.60 $\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2x\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

```
[Out] (2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/a^(3/2)
```

Rubi [A] time = 0.0676333, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1922, 1904, 206}

$$\frac{2x\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/a^(3/2)
```

Rule 1922

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q +
b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n
 - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, -((n - q)*(2*p + 3)
)]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} + \frac{\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{a}$$

$$= \frac{2x\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{a}$$

$$= \frac{2x\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}} - \frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.123031, size = 109, normalized size = 1.16

$$\frac{x\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}x\left(-2ac+b^2+bcx\right)}{a^{3/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[a]*x*(b^2 - 2*a*c + b*c*x) + (b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]
*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(a^(3/2)*(-b^2 + 4
*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.006, size = 164, normalized size = 1.7

$$\frac{x^3\left(cx^2+bx+a\right)}{4\,ac-b^2}\left(4\,a^{5/2}c-2\,a^{3/2}xbc-2\,a^{3/2}b^2-4\,\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)\sqrt{cx^2+bx+a}a^2c+\ln\left(\frac{1}{x}\left(2\,a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b*x+a)*(4*a^(5/2)*c-2*a^(3/2)*x*b*c-2*a^(3/2)*b^2-4*ln((2*a+b*x+
2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a^2*c+ln((2*a+b*x+2*a
^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a*b^2)/(c*x^4+b*x^3+a*x^
2)^(3/2)/a^(5/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [B] time = 2.06535, size = 869, normalized size = 9.24

$$\left[\frac{\left(\left(b^2c - 4\,ac^2\right)x^3 + \left(b^3 - 4\,abc\right)x^2 + \left(ab^2 - 4\,a^2c\right)x\right)\sqrt{a}\log\left(-\frac{8\,abx^2 + \left(b^2 + 4\,ac\right)x^3 + 8\,a^2x - 4\,\sqrt{cx^4 + bx^3 + ax^2}(bx + 2\,a)\sqrt{a}}{x^3}\right) + 4\,\sqrt{cx^4 + bx^3}}{2\left(\left(a^2b^2c - 4\,a^3c^2\right)x^3 + \left(a^2b^3 - 4\,a^3bc\right)x^2 + \left(a^3b^2 - 4\,a^4c\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*s
qrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3
 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c*
x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b*c)*
x^2 + (a^3*b^2 - 4*a^4*c)*x), (((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2
 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*
x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 -
4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.61 $\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2\left(b^2-4ac\right)}+\frac{3b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}+\frac{2\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (
(3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a^2*(b^2 - 4*a*c)*x^2) + (3*b
*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(5/
2))
```

Rubi [A] time = 0.163171, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1924, 1951, 12, 1904, 206}

$$-\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2\left(b^2-4ac\right)}+\frac{3b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}+\frac{2\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (
(3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a^2*(b^2 - 4*a*c)*x^2) + (3*b
*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(5/
2))
```

Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
 && LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx &= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int \frac{-\frac{3b^2}{2}+4ac-bcx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}\\
&= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2}+\frac{2\int -\frac{3b\left(b^2-4ac\right)}{4\sqrt{ax^2+bx^3+cx^4}}\,dx}{a^2\left(b^2-4ac\right)}\\
&= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2}-\frac{(3b)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a^2}\\
&= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2}+\frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\right.}{a^2}\\
&= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{a^2\left(b^2-4ac\right)x^2}+\frac{3b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx}}\right.}{2a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.105634, size = 138, normalized size = 0.96

$$\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx-8c^2x^2\right)+3b^2x(b+cx)\right)-3bx\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2a^{5/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[a]*(-4*a^2*c + 3*b^2*x*(b + c*x) + a*(b^2 - 10*b*c*x - 8*c^2*x^2))
- 3*b*(b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + x*(b + c*x)])])/(2*a^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x
))])
```

Maple [A] time = 0.006, size = 201, normalized size = 1.4

$$-\frac{x^2\left(cx^2+bx+a\right)}{8ac-2b^2}\left(16a^{5/2}x^2c^2-6a^{3/2}x^2b^2c+20a^{5/2}xbc-6a^{3/2}xb^3-12\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)\sqrt{cx^2+bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/2*x^2*(c*x^2+b*x+a)*(16*a^(5/2)*x^2*c^2-6*a^(3/2)*x^2*b^2*c+20*a^(5/2)*x
*b*c-6*a^(3/2)*x*b^3-12*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^
2+b*x+a)^(1/2)*x*a^2*b*c+3*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c
*x^2+b*x+a)^(1/2)*x*a*b^3+8*a^(7/2)*c-2*a^(5/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3
/2)/a^(7/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(cx^4 + bx^3 + ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A] time = 1.87612, size = 1041, normalized size = 7.23

$$\left[\frac{3\left(\left(b^3c - 4\,abc^2\right)x^4 + \left(b^4 - 4\,ab^2c\right)x^3 + \left(ab^3 - 4\,a^2bc\right)x^2\right)\sqrt{a}\log\left(-\frac{8\,abx^2 + \left(b^2 + 4\,ac\right)x^3 + 8\,a^2x + 4\,\sqrt{cx^4 + bx^3 + ax^2}(bx + 2\,a)\sqrt{a}}{x^3}\right) - 4\,\sqrt{\ }}{4\left(\left(a^3b^2c - 4\,a^4c^2\right)x^4 + \left(a^3b^3 - 4\,a^4bc\right)x^3 + \left(a^4b^2 - 4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b*c
)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2 -
 4*a^5*c)*x^2), -1/2*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 +
(a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b
*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b
*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2
 - 4*a^5*c)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.62 $\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3\left(b^2-4ac\right)}-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}+\frac{\cdots}{ax\left(b\cdots\right.}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^3) +
 (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^3*(b^2 - 4*a*c)*x^2
) - (3*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(8*a^(7/2))
```

Rubi [A] time = 0.286989, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1907, 1951, 12, 1904, 206}

$$\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3\left(b^2-4ac\right)}-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}+\frac{\cdots}{ax\left(b\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^3) +
 (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^3*(b^2 - 4*a*c)*x^2
) - (3*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(8*a^(7/2))
```

Rule 1907

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> -Simp[(x^(-q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(
2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q
)*(p + 1)*(b^2 - 4*a*c)), Int[(((p*q + 1)*(b^2 - 2*a*c) + (n - q)*(p + 1)*(
b^2 - 4*a*c) + b*c*(p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^(p + 1))/x^q, x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r
, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && LtQ[p,
 -1]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-2\left(b^2-2ac\right)+\frac{1}{2}\left(-b^2+4ac\right)-2bcx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{\int \frac{-\frac{1}{4}b\left(15b^2-52ac\right)-\frac{1}{2}c\left(5b^2-\right.}{x\sqrt{ax^2+bx^3+cx^4}}}{a^2\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+}}{4a^3\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+}}{4a^3\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+}}{4a^3\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b\left(15b^2-52ac\right)\sqrt{ax^2+}}{4a^3\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.166795, size = 181, normalized size = 0.87

$$-\frac{2\sqrt{a}\left(2a^2\left(b^2+10bcx-12c^2x^2\right)-8a^3c+abx\left(-5b^2+62bcx+52c^2x^2\right)-15b^3x^2(b+cx)\right)+3x^2\left(16a^2c^2-24ab^2c+5b^4\right.}{8a^{7/2}x\left(b^2-4ac\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

```
[Out] -(2*Sqrt[a]*(-8*a^3*c - 15*b^3*x^2*(b + c*x) + 2*a^2*(b^2 + 10*b*c*x - 12*c
^2*x^2) + a*b*x*(-5*b^2 + 62*b*c*x + 52*c^2*x^2)) + 3*(5*b^4 - 24*a*b^2*c +
 16*a^2*c^2)*x^2*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[
a + x*(b + c*x)]])])/(8*a^(7/2)*(b^2 - 4*a*c)*x*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.007, size = 292, normalized size = 1.4

$$-\frac{x\left(cx^2+bx+a\right)}{32\,ac-8\,b^2}\left(48\,a^{7/2}x^2c^2-104\,a^{5/2}x^3bc^2+16\,a^{9/2}c-40\,a^{7/2}xbc-124\,a^{5/2}x^2b^2c+30\,a^{3/2}x^3b^3c-4\,a^{7/2}b^2+10\,a^{5/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/8*x*(c*x^2+b*x+a)*(48*a^(7/2)*x^2*c^2-104*a^(5/2)*x^3*b*c^2+16*a^(9/2)*c
-40*a^(7/2)*x*b*c-124*a^(5/2)*x^2*b^2*c+30*a^(3/2)*x^3*b^3*c-4*a^(7/2)*b^2+
10*a^(5/2)*x*b^3+30*a^(3/2)*x^2*b^4-48*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^
(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a^3*c^2+72*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b
*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a^2*b^2*c-15*ln((2*a+b*x+2*a^(1/2)*
(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a*b^4)/(c*x^4+b*x^3+a*x^2)^
(3/2)/a^(9/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2), x)
```

Fricas [A] time = 2.20791, size = 1328, normalized size = 6.35

$$\left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^5+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^4+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x^3\right)\sqrt{a}\log\left(-\frac{8\,abx^2+(}{}\right.}{16\left(\left(a^4b^2c-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^4 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(a)*lo
g(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)
*(b*x + 2*a)*sqrt(a))/x^3) + 4*(2*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*a^2*
b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^
3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*
b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*x^3), 1/8*(3*((5*b^4*c - 24*a*b^
2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^4 + (5*a*b^
4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(2*a^3*b^2 -
 8*a^4*c - (15*a*b^3*c - 52*a^2*b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*
a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^
```

```
4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*
x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^2 + bx^3 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.63 $\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{24a^4x^2\left(b^2-4ac\right)}+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+c}}{3a^2x^4\left(b^2-4ac\right)}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4])
 - ((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*a^2*(b^2 - 4*a*c)*x^4)
 + (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a^3*(b^2 - 4*a*c)
*x^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(24*a^4*(b^2 - 4*a*c)*x^2) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(x*(2*a + b*x))
/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(9/2))
```

Rubi [A] time = 0.452449, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1924, 1951, 12, 1904, 206}

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{24a^4x^2\left(b^2-4ac\right)}+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+c}}{3a^2x^4\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4])
 - ((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*a^2*(b^2 - 4*a*c)*x^4)
 + (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a^3*(b^2 - 4*a*c)
*x^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(24*a^4*(b^2 - 4*a*c)*x^2) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(x*(2*a + b*x))
/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(9/2))
```

Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
 && LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

```
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-\frac{7b^2}{2}+8ac-3bcx}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{2\int \frac{-\frac{1}{4}b\left(35b^2-116ac\right)-c\left(\cdots\right.}{x^2\sqrt{ax^2+bx^3+\cdots}}}{3a^2\left(b^2-4a\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b\left(35b^2-116ac\right)\sqrt{a\cdots}}{12a^3\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b\left(35b^2-116ac\right)\sqrt{a\cdots}}{12a^3\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b\left(35b^2-116ac\right)\sqrt{a\cdots}}{12a^3\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b\left(35b^2-116ac\right)\sqrt{a\cdots}}{12a^3\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b\left(35b^2-116ac\right)\sqrt{a\cdots}}{12a^3\left(b^2-4\cdots\right.}$$

Mathematica [A] time = 0.265435, size = 225, normalized size = 0.83

$$\frac{2\sqrt{a}\left(8a^3\left(b^2+7bcx+16c^2x^2\right)+2a^2x\left(-86b^2cx-7b^3+244bc^2x^2+128c^3x^3\right)-32a^4c+5ab^2x^2\left(7b^2-106bcx-92c^2x^2\right)\cdots\right.}{48a^{9/2}x^2\left(4ac-b^2\right)\sqrt{x^2(a+x(b+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*Sqrt[a]*(-32*a^4*c + 105*b^4*x^3*(b + c*x) + 5*a*b^2*x^2*(7*b^2 - 106*b*
c*x - 92*c^2*x^2) + 8*a^3*(b^2 + 7*b*c*x + 16*c^2*x^2) + 2*a^2*x*(-7*b^3 -
86*b^2*c*x + 244*b*c^2*x^2 + 128*c^3*x^3)) - 15*b*(7*b^4 - 40*a*b^2*c + 48*
a^2*c^2)*x^3*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
x*(b + c*x)])])/(48*a^(9/2)*(-b^2 + 4*a*c)*x^2*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.009, size = 340, normalized size = 1.3

$$-\frac{cx^2+bx+a}{192\,ac-48\,b^2}\left(-512\,a^{7/2}x^4c^3+920\,a^{5/2}x^4b^2c^2-210\,a^{3/2}x^4b^4c+720\,\sqrt{cx^2+bx+a}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/48*(c*x^2+b*x+a)*(-512*a^(7/2)*x^4*c^3+920*a^(5/2)*x^4*b^2*c^2-210*a^(3/
2)*x^4*b^4*c+720*(c*x^2+b*x+a)^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1
/2))/x)*x^3*a^3*b*c^2-600*(c*x^2+b*x+a)^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+
b*x+a)^(1/2))/x)*x^3*a^2*b^3*c+105*(c*x^2+b*x+a)^(1/2)*ln((2*a+b*x+2*a^(1/2
)*(c*x^2+b*x+a)^(1/2))/x)*x^3*a*b^5-976*a^(7/2)*x^3*b*c^2+1060*a^(5/2)*x^3*
b^3*c-210*a^(3/2)*x^3*b^5-256*a^(9/2)*x^2*c^2+344*a^(7/2)*x^2*b^2*c-70*a^(5
/2)*x^2*b^4-112*a^(9/2)*x*b*c+28*a^(7/2)*x*b^3+64*a^(11/2)*c-16*a^(9/2)*b^2
)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(11/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

Fricas [A] time = 2.91328, size = 1550, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4
*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqr
t(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c
- 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3
```

```
*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3
- 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 +
 (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4), -1/48*(15*((7*b^5*c
- 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*
x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt
(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) +
 2*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^
4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3
*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3 - 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3
+ a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b
^2 - 4*a^7*c)*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

3.64 $\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=343

$$\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{64a^5x^2\left(b^2-4ac\right)}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{32a^4x^3\left(b^2-4ac\right)}-\frac{15\left(16a^2c^2-\right.}{}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3 + c*x^4])
 - ((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^5)
 + (b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x
^4) - ((105*b^4 - 448*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(
32*a^4*(b^2 - 4*a*c)*x^3) + (b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(64*a^5*(b^2 - 4*a*c)*x^2) - (15*(21*b^4 - 56*a*b^
2*c + 16*a^2*c^2)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4])])/(128*a^(11/2))
```

Rubi [A] time = 0.620551, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1924, 1951, 12, 1904, 206}

$$\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{64a^5x^2\left(b^2-4ac\right)}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{32a^4x^3\left(b^2-4ac\right)}-\frac{15\left(16a^2c^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```


```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3 + c*x^4])
 - ((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^5)
 + (b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x
^4) - ((105*b^4 - 448*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(
32*a^4*(b^2 - 4*a*c)*x^3) + (b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(64*a^5*(b^2 - 4*a*c)*x^2) - (15*(21*b^4 - 56*a*b^
2*c + 16*a^2*c^2)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4])])/(128*a^(11/2))
```


Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
 && LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
```

```
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-\frac{9b^2}{2}+10ac-4bcx}{x^4\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{\int \frac{-\frac{3}{4}b\left(21b^2-68ac\right)-\frac{3}{2}c(\cdots}{x^3\sqrt{ax^2+bx^3+\cdots}}}{2a^2\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2-68ac\right)\sqrt{a\cdots}}{8a^3\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2-68ac\right)\sqrt{a\cdots}}{8a^3\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2-68ac\right)\sqrt{a\cdots}}{8a^3\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2-68ac\right)\sqrt{a\cdots}}{8a^3\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2-68ac\right)\sqrt{a\cdots}}{8a^3\left(b^2-4\cdots\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b\left(21b^2-68ac\right)\sqrt{a\cdots}}{8a^3\left(b^2-4\cdots\right.}$$

Mathematica [A] time = 0.286878, size = 272, normalized size = 0.79

$$15x^4\left(240a^2b^2c^2-64a^3c^3-140ab^4c+21b^6\right)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)-2\sqrt{a}\left(-16a^4\left(b^2+6bcx+10c^2x^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[a]*(64*a^5*c + 315*b^5*x^4*(b + c*x) + 105*a*b^3*x^3*(b^2 - 18*b*c
*x - 16*c^2*x^2) - 16*a^4*(b^2 + 6*b*c*x + 10*c^2*x^2) + 8*a^3*x*(3*b^3 + 2
6*b^2*c*x + 98*b*c^2*x^2 - 60*c^3*x^3) + 2*a^2*b*x^2*(-21*b^3 - 308*b^2*c*x
 + 1352*b*c^2*x^2 + 904*c^3*x^3)) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*
c^2 - 64*a^3*c^3)*x^4*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + x*(b + c*x)])])/(128*a^(11/2)*(-b^2 + 4*a*c)*x^3*Sqrt[x^2*(a + x*(
b + c*x))])
```

Maple [A] time = 0.008, size = 446, normalized size = 1.3

$$-\frac{cx^2+bx+a}{128\,x\left(4\,ac-b^2\right)}\left(3616\,a^{7/2}x^5bc^3-3360\,a^{5/2}x^5b^3c^2+630\,a^{3/2}x^5b^5c-960\,a^{9/2}x^4c^3+5408\,a^{7/2}x^4b^2c^2-3780\,a^{5/2}x^4b^4c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/128/x*(c*x^2+b*x+a)*(3616*a^(7/2)*x^5*b*c^3-3360*a^(5/2)*x^5*b^3*c^2+630
*a^(3/2)*x^5*b^5*c-960*a^(9/2)*x^4*c^3+5408*a^(7/2)*x^4*b^2*c^2-3780*a^(5/2
)*x^4*b^4*c+630*a^(3/2)*x^4*b^6+960*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a^4*c^3-3600*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a^3*b^2*c^2+2100*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a^2*b^4*c-315*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a*b^6+1568*a^(9
/2)*x^3*b*c^2-1232*a^(7/2)*x^3*b^3*c+210*a^(5/2)*x^3*b^5-320*a^(11/2)*x^2*c
^2+416*a^(9/2)*x^2*b^2*c-84*a^(7/2)*x^2*b^4-192*a^(11/2)*x*b*c+48*a^(9/2)*x
*b^3+128*a^(13/2)*c-32*a^(11/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(13/2)/(4*
a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)
```

Fricas [A] time = 4.08513, size = 1901, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 +
 (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6 -
140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*x^5)*sqrt(a)*log(-(8*a*b*x^2
+ (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*s
qrt(a))/x^3) - 4*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*c^2 +
 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480
*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a
^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3 - 4*a^5*b*c)*x)*sqrt
(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 + (a^6*b^3 - 4*a^7*b*
c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5), 1/128*(15*((21*b^6*c - 140*a*b^4*c^2 + 2
40*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2
- 64*a^3*b*c^3)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*
c^3)*x^5)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(
-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c
- 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 27
04*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*
b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3
- 4*a^5*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 +
 (a^6*b^3 - 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\left(x^{2}\left(a+bx+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{4}+bx^{3}+ax^{2}\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)
```

3.65 $\int x^m \left(ax + bx^3 + cx^5\right) dx$

Optimal. Leaf size=37

$$\frac{ax^{m+2}}{m+2} + \frac{bx^{m+4}}{m+4} + \frac{cx^{m+6}}{m+6}$$

```
[Out] (a*x^(2 + m))/(2 + m) + (b*x^(4 + m))/(4 + m) + (c*x^(6 + m))/(6 + m)
```

Rubi [A] time = 0.0129212, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{ax^{m+2}}{m+2} + \frac{bx^{m+4}}{m+4} + \frac{cx^{m+6}}{m+6}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^(2 + m))/(2 + m) + (b*x^(4 + m))/(4 + m) + (c*x^(6 + m))/(6 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^m \left(ax + bx^3 + cx^5\right) dx &= \int \left(ax^{1+m} + bx^{3+m} + cx^{5+m}\right) dx \\ &= \frac{ax^{2+m}}{2+m} + \frac{bx^{4+m}}{4+m} + \frac{cx^{6+m}}{6+m} \end{aligned}$$

Mathematica [A] time = 0.0288071, size = 34, normalized size = 0.92

$$x^{m+2}\left(\frac{a}{m+2} + \frac{bx^2}{m+4} + \frac{cx^4}{m+6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x^(2 + m)*(a/(2 + m) + (b*x^2)/(4 + m) + (c*x^4)/(6 + m))
```

Maple [B] time = 0.003, size = 77, normalized size = 2.1

$$\frac{x^{2+m}\left(cm^2x^4 + 6\,cmx^4 + bm^2x^2 + 8\,cx^4 + 8\,bmx^2 + am^2 + 12\,bx^2 + 10\,am + 24\,a\right)}{(6+m)\,(4+m)\,(2+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^5+b*x^3+a*x),x)
```

```
[Out] x^(2+m)*(c*m^2*x^4+6*c*m*x^4+b*m^2*x^2+8*c*x^4+8*b*m*x^2+a*m^2+12*b*x^2+10*
a*m+24*a)/(6+m)/(4+m)/(2+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31314, size = 161, normalized size = 4.35

$$\frac{\left(\left(cm^2+6\,cm+8\,c\right)x^6+\left(bm^2+8\,bm+12\,b\right)x^4+\left(am^2+10\,am+24\,a\right)x^2\right)x^m}{m^3+12\,m^2+44\,m+48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 6*c*m + 8*c)*x^6 + (b*m^2 + 8*b*m + 12*b)*x^4 + (a*m^2 + 10*a*m +
 24*a)*x^2)*x^m/(m^3 + 12*m^2 + 44*m + 48)
```

Sympy [A] time = 1.09027, size = 280, normalized size = 7.57

$$\begin{cases} -\frac{a}{4x^4}-\frac{b}{2x^2}+c\log{(x)} \\ -\frac{a}{2x^2}+b\log{(x)}+\frac{cx^2}{2} \\ a\log{(x)}+\frac{bx^2}{2}+\frac{cx^4}{4} \\ \frac{am^2x^2x^m}{m^3+12m^2+44m+48}+\frac{10amx^2x^m}{m^3+12m^2+44m+48}+\frac{24ax^2x^m}{m^3+12m^2+44m+48}+\frac{bm^2x^4x^m}{m^3+12m^2+44m+48}+\frac{8bmx^4x^m}{m^3+12m^2+44m+48}+\frac{12bx^4x^m}{m^3+12m^2+44m+48}+\frac{cm^2x^6x^m}{m^3+12m^2+4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**5+b*x**3+a*x),x)
```

```
[Out] Piecewise((-a/(4*x**4) - b/(2*x**2) + c*log(x), Eq(m, -6)), (-a/(2*x**2) +
b*log(x) + c*x**2/2, Eq(m, -4)), (a*log(x) + b*x**2/2 + c*x**4/4, Eq(m, -2)
), (a*m**2*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + 10*a*m*x**2*x**m/(m**3
+ 12*m**2 + 44*m + 48) + 24*a*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + b*m*
*2*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + 8*b*m*x**4*x**m/(m**3 + 12*m**2
 + 44*m + 48) + 12*b*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + c*m**2*x**6*x
**m/(m**3 + 12*m**2 + 44*m + 48) + 6*c*m*x**6*x**m/(m**3 + 12*m**2 + 44*m +
 48) + 8*c*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48), True))
```

<u>**Giac**</u> **[B]** time = 1.11225, size = 144, normalized size = 3.89

$$\frac{cm^2x^6x^m + 6\,cmx^6x^m + bm^2x^4x^m + 8\,cx^6x^m + 8\,bmx^4x^m + am^2x^2x^m + 12\,bx^4x^m + 10\,amx^2x^m + 24\,ax^2x^m}{m^3 + 12\,m^2 + 44\,m + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] (c*m^2*x^6*x^m + 6*c*m*x^6*x^m + b*m^2*x^4*x^m + 8*c*x^6*x^m + 8*b*m*x^4*x^
m + a*m^2*x^2*x^m + 12*b*x^4*x^m + 10*a*m*x^2*x^m + 24*a*x^2*x^m)/(m^3 + 12
*m^2 + 44*m + 48)
```

3.66 $\int x^2 \left(ax + bx^3 + cx^5\right) dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Rubi [A] time = 0.0078675, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(ax + bx^3 + cx^5\right) dx &= \int \left(ax^3 + bx^5 + cx^7\right) dx\\ &= \frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}\end{aligned}$$

Mathematica [A] time = 0.0019507, size = 25, normalized size = 1.

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Maple [A] time = 0.002, size = 20, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4*a*x^4+1/6*b*x^6+1/8*c*x^8
```

Maxima [A] time = 1.12666, size = 26, normalized size = 1.04

$$\frac{1}{8}\,cx^8+\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

Fricas [A] time = 1.0509, size = 47, normalized size = 1.88

$$\frac{1}{8}x^8c+\frac{1}{6}x^6b+\frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c + 1/6*x^6*b + 1/4*x^4*a
```

Sympy [A] time = 0.056171, size = 19, normalized size = 0.76

$$\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**5+b*x**3+a*x),x)
```

```
[Out] a*x**4/4 + b*x**6/6 + c*x**8/8
```

Giac [A] time = 1.09845, size = 26, normalized size = 1.04

$$\frac{1}{8}\,cx^8+\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

3.67 $\int x \left(ax+bx^3+cx^5\right) dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rubi [A] time = 0.0072531, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {14}

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax+bx^3+cx^5\right)dx &= \int \left(ax^2+bx^4+cx^6\right)dx\\ &= \frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0018573, size = 25, normalized size = 1.

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Maxima [A] time = 1.072, size = 26, normalized size = 1.04

$$\frac{1}{7}cx^7 + \frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A] time = 1.08682, size = 47, normalized size = 1.88

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c + 1/5*x^5*b + 1/3*x^3*a
```

Sympy [A] time = 0.056432, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**5+b*x**3+a*x),x)
```

```
[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

Giac [A] time = 1.0907, size = 26, normalized size = 1.04

$$\frac{1}{7}cx^7 + \frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

3.68 $\int \left(ax + bx^3 + cx^5\right) dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi [A] time = 0.0044085, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[a*x + b*x^3 + c*x^5,x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi steps

$$\int \left(ax + bx^3 + cx^5\right) dx = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Mathematica [A] time = 0.0000472, size = 25, normalized size = 1.

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x + b*x^3 + c*x^5,x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^5+b*x^3+a*x,x)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Maxima [A] time = 1.11375, size = 26, normalized size = 1.04

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Fricas [A] time = 1.04867, size = 47, normalized size = 1.88

$$\frac{1}{6}x^6c + \frac{1}{4}x^4b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c + 1/4*x^4*b + 1/2*x^2*a
```

Sympy [A] time = 0.055644, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**5+b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

Giac [A] time = 1.10021, size = 26, normalized size = 1.04

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

3.69 $\int \frac{ax+bx^3+cx^5}{x} dx$

Optimal. Leaf size=20

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi [A] time = 0.0056522, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)/x,x]

[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+bx^3+cx^5}{x} dx &= \int \left(a+bx^2+cx^4\right) dx\\ &= ax + \frac{bx^3}{3} + \frac{cx^5}{5}\end{aligned}$$

Mathematica [A] time = 0.0011684, size = 20, normalized size = 1.

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)/x,x]

[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Maple [A] time = 0., size = 17, normalized size = 0.9

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)/x,x)
```

```
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Maxima [A] time = 1.13096, size = 22, normalized size = 1.1

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Fricas [A] time = 1.22959, size = 39, normalized size = 1.95

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="fricas")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Sympy [A] time = 0.055914, size = 15, normalized size = 0.75

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)/x,x)
```

```
[Out] a*x + b*x**3/3 + c*x**5/5
```

Giac [A] time = 1.09995, size = 22, normalized size = 1.1

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

3.70 $\int \frac{ax+bx^3+cx^5}{x^2} \, dx$

Optimal. Leaf size=21

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rubi [A] time = 0.0081022, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)/x^2,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax+bx^3+cx^5}{x^2} \, dx &= \int \left(\frac{a}{x} + bx + cx^3\right) dx \\ &= \frac{bx^2}{2} + \frac{cx^4}{4} + a\log(x) \end{aligned}$$

Mathematica [A] time = 0.001807, size = 21, normalized size = 1.

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)/x^2,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Maple [A] time = 0.003, size = 18, normalized size = 0.9

$$\frac{bx^2}{2} + \frac{cx^4}{4} + a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)/x^2,x)

[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Maxima [A] time = 1.14202, size = 23, normalized size = 1.1

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="maxima")

[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Fricas [A] time = 1.22428, size = 46, normalized size = 2.19

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="fricas")

[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Sympy [A] time = 0.089078, size = 17, normalized size = 0.81

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)/x**2,x)

[Out] a*log(x) + b*x**2/2 + c*x**4/4
```

Giac [A] time = 1.1209, size = 27, normalized size = 1.29

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2 + \frac{1}{2}a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="giac")

[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

3.71 $\int \frac{ax+bx^3+cx^5}{x^3} dx$

Optimal. Leaf size=18

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rubi [A] time = 0.0068819, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)/x^3,x]

[Out] -(a/x) + b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+bx^3+cx^5}{x^3} dx &= \int \left(b+\frac{a}{x^2}+cx^2\right) dx\\ &= -\frac{a}{x}+bx+\frac{cx^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0020158, size = 18, normalized size = 1.

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)/x^3,x]

[Out] -(a/x) + b*x + (c*x^3)/3
```

Maple [A] time = 0.003, size = 17, normalized size = 0.9

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)/x^3,x)
```

```
[Out] -a/x+b*x+1/3*c*x^3
```

Maxima [A] time = 1.11163, size = 22, normalized size = 1.22

$$\frac{1}{3}\,cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

Fricas [A] time = 1.19098, size = 42, normalized size = 2.33

$$\frac{cx^4 + 3\,bx^2 - 3\,a}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

Sympy [A] time = 0.247622, size = 12, normalized size = 0.67

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)/x**3,x)
```

```
[Out] -a/x + b*x + c*x**3/3
```

Giac [A] time = 1.09081, size = 22, normalized size = 1.22

$$\frac{1}{3}\,cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

3.72 $\int x^m \left(ax+bx^3+cx^5\right)^2 dx$

Optimal. Leaf size=76

$$\frac{a^2x^{m+3}}{m+3}+\frac{x^{m+7}\left(2ac+b^2\right)}{m+7}+\frac{2abx^{m+5}}{m+5}+\frac{2bcx^{m+9}}{m+9}+\frac{c^2x^{m+11}}{m+11}$$

```
[Out] (a^2*x^(3 + m))/(3 + m) + (2*a*b*x^(5 + m))/(5 + m) + ((b^2 + 2*a*c)*x^(7 +
 m))/(7 + m) + (2*b*c*x^(9 + m))/(9 + m) + (c^2*x^(11 + m))/(11 + m)
```

Rubi [A] time = 0.0458617, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1585, 1108}

$$\frac{a^2x^{m+3}}{m+3}+\frac{x^{m+7}\left(2ac+b^2\right)}{m+7}+\frac{2abx^{m+5}}{m+5}+\frac{2bcx^{m+9}}{m+9}+\frac{c^2x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^(3 + m))/(3 + m) + (2*a*b*x^(5 + m))/(5 + m) + ((b^2 + 2*a*c)*x^(7 +
 m))/(7 + m) + (2*b*c*x^(9 + m))/(9 + m) + (c^2*x^(11 + m))/(11 + m)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^m\left(ax+bx^3+cx^5\right)^2 dx &= \int x^{2+m}\left(a+bx^2+cx^4\right)^2 dx\\ &= \int \left(a^2x^{2+m}+2abx^{4+m}+\left(b^2+2ac\right)x^{6+m}+2bcx^{8+m}+c^2x^{10+m}\right) dx\\ &= \frac{a^2x^{3+m}}{3+m}+\frac{2abx^{5+m}}{5+m}+\frac{\left(b^2+2ac\right)x^{7+m}}{7+m}+\frac{2bcx^{9+m}}{9+m}+\frac{c^2x^{11+m}}{11+m}\end{aligned}$$

Mathematica [A] time = 0.0787956, size = 69, normalized size = 0.91

$$x^{m+3}\left(\frac{a^2}{m+3}+\frac{x^4\left(2ac+b^2\right)}{m+7}+\frac{2abx^2}{m+5}+\frac{2bcx^6}{m+9}+\frac{c^2x^8}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] x^(3 + m)*(a^2/(3 + m) + (2*a*b*x^2)/(5 + m) + ((b^2 + 2*a*c)*x^4)/(7 + m)
+ (2*b*c*x^6)/(9 + m) + (c^2*x^8)/(11 + m))
```

Maple [B] time = 0.006, size = 300, normalized size = 4.

$$\frac{x^{3+m}\left(c^2m^4x^8+24\,c^2m^3x^8+2\,bcm^4x^6+206\,c^2m^2x^8+52\,bcm^3x^6+744\,c^2mx^8+2\,acm^4x^4+b^2m^4x^4+472\,bcm^2x^6+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] x^(3+m)*(c^2*m^4*x^8+24*c^2*m^3*x^8+2*b*c*m^4*x^6+206*c^2*m^2*x^8+52*b*c*m^
3*x^6+744*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+472*b*c*m^2*x^6+945*c^2*x^8+5
6*a*c*m^3*x^4+28*b^2*m^3*x^4+1772*b*c*m*x^6+2*a*b*m^4*x^2+548*a*c*m^2*x^4+2
74*b^2*m^2*x^4+2310*b*c*x^6+60*a*b*m^3*x^2+2184*a*c*m*x^4+1092*b^2*m*x^4+a^
2*m^4+640*a*b*m^2*x^2+2970*a*c*x^4+1485*b^2*x^4+32*a^2*m^3+2820*a*b*m*x^2+3
74*a^2*m^2+4158*a*b*x^2+1888*a^2*m+3465*a^2)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.4098, size = 602, normalized size = 7.92

$$\frac{\left(\left(c^2m^4+24\,c^2m^3+206\,c^2m^2+744\,c^2m+945\,c^2\right)x^{11}+2\left(bcm^4+26\,bcm^3+236\,bcm^2+886\,bcm+1155\,bc\right)x^9+\left(\left(b\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 24*c^2*m^3 + 206*c^2*m^2 + 744*c^2*m + 945*c^2)*x^11 + 2*(b*c*m
^4 + 26*b*c*m^3 + 236*b*c*m^2 + 886*b*c*m + 1155*b*c)*x^9 + ((b^2 + 2*a*c)*
m^4 + 28*(b^2 + 2*a*c)*m^3 + 274*(b^2 + 2*a*c)*m^2 + 1485*b^2 + 2970*a*c +
1092*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 30*a*b*m^3 + 320*a*b*m^2 + 1410*a*
b*m + 2079*a*b)*x^5 + (a^2*m^4 + 32*a^2*m^3 + 374*a^2*m^2 + 1888*a^2*m + 34
65*a^2)*x^3)*x^m/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

Sympy [A] time = 3.99035, size = 1377, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Piecewise((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) - b
*c/x**2 + c**2*log(x), Eq(m, -11)), (-a**2/(6*x**6) - a*b/(2*x**4) - a*c/x*
*2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2, Eq(m, -9)), (-a**2/(4*x**4
) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c**2*x**4/4, Eq(m, -
7)), (-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*x**2/2 + b*c*x**4/2 +
 c**2*x**6/6, Eq(m, -5)), (a**2*log(x) + a*b*x**2 + a*c*x**4/2 + b**2*x**4/
4 + b*c*x**6/3 + c**2*x**8/8, Eq(m, -3)), (a**2*m**4*x**3*x**m/(m**5 + 35*m
**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 32*a**2*m**3*x**3*x**m/(m**5
 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 374*a**2*m**2*x**3*x*
*m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1888*a**2*m*x
**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 3465*a*
*2*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2*a
*b*m**4*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
+ 60*a*b*m**3*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 1
0395) + 640*a*b*m**2*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 912
9*m + 10395) + 2820*a*b*m*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
+ 9129*m + 10395) + 4158*a*b*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m*
*2 + 9129*m + 10395) + 2*a*c*m**4*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 30
10*m**2 + 9129*m + 10395) + 56*a*c*m**3*x**7*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 548*a*c*m**2*x**7*x**m/(m**5 + 35*m**4 +
470*m**3 + 3010*m**2 + 9129*m + 10395) + 2184*a*c*m*x**7*x**m/(m**5 + 35*m*
*4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2970*a*c*x**7*x**m/(m**5 + 35
*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + b**2*m**4*x**7*x**m/(m**5
+ 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 28*b**2*m**3*x**7*x**m
/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 274*b**2*m**2*x
**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1092*b*
*2*m*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1
485*b**2*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
 + 2*b*c*m**4*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 1
0395) + 52*b*c*m**3*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129
*m + 10395) + 472*b*c*m**2*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
 + 9129*m + 10395) + 1772*b*c*m*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010
*m**2 + 9129*m + 10395) + 2310*b*c*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3
010*m**2 + 9129*m + 10395) + c**2*m**4*x**11*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 24*c**2*m**3*x**11*x**m/(m**5 + 35*m**4 +
 470*m**3 + 3010*m**2 + 9129*m + 10395) + 206*c**2*m**2*x**11*x**m/(m**5 +
35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 744*c**2*m*x**11*x**m/(m
**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 945*c**2*x**11*x**
m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395), True))
```

Giac [B] time = 1.1273, size = 539, normalized size = 7.09

$$\frac{c^2m^4x^{11}x^m + 24\,c^2m^3x^{11}x^m + 2\,bcm^4x^9x^m + 206\,c^2m^2x^{11}x^m + 52\,bcm^3x^9x^m + 744\,c^2mx^{11}x^m + b^2m^4x^7x^m + 2\,acm^4x^7x^m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x^11*x^m + 24*c^2*m^3*x^11*x^m + 2*b*c*m^4*x^9*x^m + 206*c^2*m^2*x
^11*x^m + 52*b*c*m^3*x^9*x^m + 744*c^2*m*x^11*x^m + b^2*m^4*x^7*x^m + 2*a*c
*m^4*x^7*x^m + 472*b*c*m^2*x^9*x^m + 945*c^2*x^11*x^m + 28*b^2*m^3*x^7*x^m
+ 56*a*c*m^3*x^7*x^m + 1772*b*c*m*x^9*x^m + 2*a*b*m^4*x^5*x^m + 274*b^2*m^2
```

```
*x^7*x^m + 548*a*c*m^2*x^7*x^m + 2310*b*c*x^9*x^m + 60*a*b*m^3*x^5*x^m + 10
92*b^2*m*x^7*x^m + 2184*a*c*m*x^7*x^m + a^2*m^4*x^3*x^m + 640*a*b*m^2*x^5*x
^m + 1485*b^2*x^7*x^m + 2970*a*c*x^7*x^m + 32*a^2*m^3*x^3*x^m + 2820*a*b*m*
x^5*x^m + 374*a^2*m^2*x^3*x^m + 4158*a*b*x^5*x^m + 1888*a^2*m*x^3*x^m + 346
5*a^2*x^3*x^m)/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

3.73 $\int x^2 \left(ax+bx^3+cx^5\right)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^
2*x^13)/13
```

Rubi [A] time = 0.0360609, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1585, 1108}

$$\frac{a^2x^5}{5}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^
2*x^13)/13
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax+bx^3+cx^5\right)^2 dx &= \int x^4\left(a+bx^2+cx^4\right)^2 dx\\ &= \int \left(a^2x^4+2abx^6+\left(b^2+2ac\right)x^8+2bcx^{10}+c^2x^{12}\right) dx\\ &= \frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.0070494, size = 54, normalized size = 1.

$$\frac{a^2x^5}{5}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^
2*x^13)/13
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^5}{5}+\frac{2\,abx^7}{7}+\frac{\left(2\,ac+b^2\right)x^9}{9}+\frac{2\,bcx^{11}}{11}+\frac{c^2x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^2*x^13
```

Maxima [A] time = 1.11908, size = 59, normalized size = 1.09

$$\frac{1}{13}\,c^2x^{13}+\frac{2}{11}\,bcx^{11}+\frac{1}{9}\left(b^2+2\,ac\right)x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x^7 + 1/5*a
^2*x^5
```

Fricas [A] time = 1.01494, size = 117, normalized size = 2.17

$$\frac{1}{13}x^{13}c^2+\frac{2}{11}x^{11}cb+\frac{1}{9}x^9b^2+\frac{2}{9}x^9ca+\frac{2}{7}x^7ba+\frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*c^2 + 2/11*x^11*c*b + 1/9*x^9*b^2 + 2/9*x^9*c*a + 2/7*x^7*b*a + 1
/5*x^5*a^2
```

Sympy [A] time = 0.069509, size = 51, normalized size = 0.94

$$\frac{a^2x^5}{5}+\frac{2abx^7}{7}+\frac{2bcx^{11}}{11}+\frac{c^2x^{13}}{13}+x^9\left(\frac{2ac}{9}+\frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + 2*b*c*x**11/11 + c**2*x**13/13 + x**9*(2*a*c/9
 + b**2/9)
```

Giac [A] time = 1.08607, size = 62, normalized size = 1.15

$$\frac{1}{13}\,c^2x^{13} + \frac{2}{11}\,bcx^{11} + \frac{1}{9}\,b^2x^9 + \frac{2}{9}\,acx^9 + \frac{2}{7}\,abx^7 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 2/7*a*b*x^7 + 1
/5*a^2*x^5
```

3.74 $\int x\left(ax+bx^3+cx^5\right)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/8 + (b*c*x^10)/5 + (c^2*x^1
2)/12
```

Rubi [A] time = 0.0544296, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1585, 1114, 631}

$$\frac{a^2x^4}{4}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/8 + (b*c*x^10)/5 + (c^2*x^1
2)/12
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(ax+bx^3+cx^5\right)^2\,dx&=\int x^3\left(a+bx^2+cx^4\right)^2\,dx\\&=\frac{1}{2}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^2\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^2x+2abx^2+\left(b^2+2ac\right)x^3+2bcx^4+c^2x^5\right)dx,x,x^2\right)\\&=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}\end{aligned}$$

Mathematica [A] time = 0.0092113, size = 48, normalized size = 0.89

$$\frac{1}{120}x^4\left(30a^2+15x^4\left(2ac+b^2\right)+40abx^2+24bcx^6+10c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x^4*(30*a^2 + 40*a*b*x^2 + 15*(b^2 + 2*a*c)*x^4 + 24*b*c*x^6 + 10*c^2*x^8)
)/120
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{\left(2\,ac+b^2\right)x^8}{8}+\frac{bcx^{10}}{5}+\frac{c^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12
```

Maxima [A] time = 1.0896, size = 59, normalized size = 1.09

$$\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\,\left(b^2+2\,ac\right)x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^6 + 1/4*a^
2*x^4
```

Fricas [A] time = 1.1233, size = 116, normalized size = 2.15

$$\frac{1}{12}x^{12}c^2+\frac{1}{5}x^{10}cb+\frac{1}{8}x^8b^2+\frac{1}{4}x^8ca+\frac{1}{3}x^6ba+\frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*c^2 + 1/5*x^10*c*b + 1/8*x^8*b^2 + 1/4*x^8*c*a + 1/3*x^6*b*a + 1/
4*x^4*a^2
```

Sympy [A] time = 0.069945, size = 46, normalized size = 0.85

$$\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{bcx^{10}}{5}+\frac{c^2x^{12}}{12}+x^8\left(\frac{ac}{4}+\frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b*c*x**10/5 + c**2*x**12/12 + x**8*(a*c/4 + b**2
/8)
```

Giac [A] time = 1.08953, size = 62, normalized size = 1.15

$$\frac{1}{12}c^2x^{12} + \frac{1}{5}bcx^{10} + \frac{1}{8}b^2x^8 + \frac{1}{4}acx^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 1/3*a*b*x^6 + 1/
4*a^2*x^4
```

3.75 $\int \left(ax+bx^3+cx^5\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Rubi [A] time = 0.0263768, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1594, 1108}

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \left(ax+bx^3+cx^5\right)^2 \, dx &= \int x^2\left(a+bx^2+cx^4\right)^2 \, dx\\
&= \int \left(a^2x^2+2abx^4+\left(b^2+2ac\right)x^6+2bcx^8+c^2x^{10}\right) dx\\
&= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}
\end{aligned}$$

Mathematica [A] time = 0.0075766, size = 54, normalized size = 1.

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Maple [A] time = 0.002, size = 45, normalized size = 0.8

$$\frac{a^2x^3}{3}+\frac{2\,abx^5}{5}+\frac{\left(2\,ac+b^2\right)x^7}{7}+\frac{2\,bcx^9}{9}+\frac{c^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Maxima [A] time = 1.12043, size = 65, normalized size = 1.2

$$\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\,b^2x^7+\frac{1}{3}\,a^2x^3+\frac{2}{35}\left(5\,cx^7+7\,bx^5\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 1/3*a^2*x^3 + 2/35*(5*c*x^7 + 7
*b*x^5)*a
```

Fricas [A] time = 1.11846, size = 115, normalized size = 2.13

$$\frac{1}{11}x^{11}c^2+\frac{2}{9}x^9cb+\frac{1}{7}x^7b^2+\frac{2}{7}x^7ca+\frac{2}{5}x^5ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2 + 2/9*x^9*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 2/5*x^5*b*a + 1/3
*x^3*a^2
```

Sympy [A] time = 0.069614, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{2bcx^9}{9}+\frac{c^2x^{11}}{11}+x^7\left(\frac{2ac}{7}+\frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7 +
 b**2/7)
```

Giac [A] time = 1.09849, size = 62, normalized size = 1.15

$$\frac{1}{11}c^2x^{11} + \frac{2}{9}bcx^9 + \frac{1}{7}b^2x^7 + \frac{2}{7}acx^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/3
*a^2*x^3
```

3.76 $\int \frac{\left(ax+bx^3+cx^5\right)^2}{x}\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10
)/10
```

Rubi [A] time = 0.0456916, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1585, 1107, 611}

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^2/x,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10
)/10
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax+bx^3+cx^5\right)^2}{x}\,dx &= \int x\left(a+bx^2+cx^4\right)^2\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a+bx+cx^2\right)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^2+2abx+b^2\left(1+\frac{2ac}{b^2}\right)x^2+2bcx^3+c^2x^4\right)dx,x,x^2\right)\\ &= \frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.0086665, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2\left(30a^2+10x^4\left(2ac+b^2\right)+30abx^2+15bcx^6+6c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x,x]
```

```
[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8))
/60
```

Maple [A] time = 0.002, size = 45, normalized size = 0.8

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{\left(2\,ac+b^2\right)x^6}{6}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^2/x,x)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Maxima [A] time = 1.02237, size = 59, normalized size = 1.09

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\left(b^2+2\,ac\right)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

Fricas [A] time = 1.30918, size = 109, normalized size = 2.02

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\left(b^2+2\,ac\right)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

Sympy [A] time = 0.069357, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**2/x,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2/
6)
```

Giac [A] time = 1.08928, size = 62, normalized size = 1.15

$$\frac{1}{10}c^2x^{10} + \frac{1}{4}bcx^8 + \frac{1}{6}b^2x^6 + \frac{1}{3}acx^6 + \frac{1}{2}abx^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/2
*a^2*x^2
```

3.77 $\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx$

Optimal. Leaf size=49

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rubi [A] time = 0.0272706, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1585, 1090}

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax+bx^3+cx^5\right)^2}{x^2}\,dx &= \int \left(a+bx^2+cx^4\right)^2\,dx\\ &= \int \left(a^2+2abx^2+b^2\left(1+\frac{2ac}{b^2}\right)x^4+2bcx^6+c^2x^8\right)dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.0053894, size = 49, normalized size = 1.

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A] time = 0.001, size = 42, normalized size = 0.9

$$xa^2+\frac{2\,abx^3}{3}+\frac{\left(2\,ac+b^2\right)x^5}{5}+\frac{2\,bcx^7}{7}+\frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^2/x^2,x)
```

```
[Out] x*a^2+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A] time = 1.12796, size = 55, normalized size = 1.12

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\left(b^2+2\,ac\right)x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A] time = 1.09929, size = 99, normalized size = 2.02

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\left(b^2+2\,ac\right)x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] time = 0.072256, size = 48, normalized size = 0.98

$$a^2x+\frac{2abx^3}{3}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}+x^5\left(\frac{2ac}{5}+\frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**2/x**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

Giac [A] time = 1.07832, size = 58, normalized size = 1.18

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.78 $\int \frac{x^8}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=100

$$\frac{(b^2-ac)\log(a+bx^2+cx^4)}{4c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rubi [A] time = 0.121738, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1585, 1114, 701, 634, 618, 206, 628}

$$\frac{(b^2-ac)\log(a+bx^2+cx^4)}{4c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{ax+bx^3+cx^5}\,dx &= \int \frac{x^7}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx,x,x^2\right) \\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\operatorname{Subst}\left(\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2} \\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}-\frac{\left(b\left(b^2-3ac\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}+\frac{\left(b^2-ac\right)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3} \\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^3} \\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}
\end{aligned}
$$

Mathematica [A] time = 0.0937252, size = 93, normalized size = 0.93

$$
\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx^2\left(cx^2-2b\right)}{4c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A] time = 0.005, size = 142, normalized size = 1.4

$$
\frac{x^4}{4c}-\frac{bx^2}{2c^2}-\frac{\ln\left(cx^4+bx^2+a\right)a}{4c^2}+\frac{\ln\left(cx^4+bx^2+a\right)b^2}{4c^3}+\frac{3ab}{2c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{b^3}{2c^3}\text{arcta}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4*x^4/c-1/2*b*x^2/c^2-1/4/c^2*ln(c*x^4+b*x^2+a)*a+1/4/c^3*ln(c*x^4+b*x^2+
a)*b^2+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b-
1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^4 - 2\,bx^2}{4\,c^2} - \frac{-\int \frac{(b^2-ac)x^3+abx}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 - integrate(-((b^2 - a*c)*x^3 + a*b*x)/(c*x^4 + b
*x^2 + a), x)/c^2
```

Fricas [A] time = 1.329, size = 675, normalized size = 6.75

$$\left[\frac{\left(b^2c^2 - 4\,ac^3\right)x^4 - 2\left(b^3c - 4\,abc^2\right)x^2 - \left(b^3 - 3\,abc\right)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right) + \left(b^4 - 5\,ab^2c\right.}{4\left(b^2c^3 - 4\,ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c)
*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)
*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2
*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2
)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] time = 1.89168, size = 391, normalized size = 3.91

$$-\frac{bx^2}{2c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)} - \frac{ac-b^2}{4c^3}\right)\log\left(x^2 + \frac{2a^2c - ab^2 + 8ac^3\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)} - \frac{ac-b^2}{4c^3}\right) - 2b^2c^2\left(-\frac{b\sqrt{-4}}{4\ldots}\right.}{3abc - b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c -
b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(-
b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)
/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*
a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c +
 b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(
x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*sqrt(-4*a
*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)))
/(3*a*b*c - b**3)) + x**4/(4*c)
```

Giac [A] time = 1.09998, size = 124, normalized size = 1.24

$$\frac{cx^4 - 2\,bx^2}{4\,c^2} + \frac{\left(b^2 - ac\right)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1/
2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*c^3)
```

3.79 $\int \frac{x^7}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=203

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.602493, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1585, 1122, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x + b*x^3 + c*x^5),x]
```


```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
```

```
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{ax+bx^3+cx^5}\,dx &= \int \frac{x^6}{a+bx^2+cx^4}\,dx\\
&= \frac{x^3}{3c} - \frac{\int \frac{x^2(3a+3bx^2)}{a+bx^2+cx^4}\,dx}{3c}\\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\int \frac{3ab+3(b^2-ac)x^2}{a+bx^2+cx^4}\,dx}{3c^2}\\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}\\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.172064, size = 250, normalized size = 1.23

$$\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 3
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.023, size = 467, normalized size = 2.3

$$\frac{x^3}{3c}-\frac{bx}{c^2}-\frac{\sqrt{2}\,a}{2c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,b^2}{2c^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+\ldots}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/3*x^3/c-b*x/c^2-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a+1/2/c^2*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b^3+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c^2*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^3-3\,bx}{3\,c^2}-\frac{-\int \frac{\left(b^2-ac\right)x^2+ab}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x^4 + b*x^2
 + a), x)/c^2
```

Fricas [B] time = 1.54029, size = 3217, normalized size = 15.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b
^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c
^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
```

```
) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c
^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(
1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sq
rt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^
2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*
a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

Sympy [A] time = 2.20684, size = 194, normalized size = 0.96

$$-\frac{bx}{c^2} + \mathrm{RootSum}\left(t^4\left(256a^2c^7 - 128ab^2c^6 + 16b^4c^5\right) + t^2\left(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7\right) + a^5, \left(t \mapsto t\log\left(x + -\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
+ _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a**
5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8*_t
**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c -
 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

Giac [C] time = 2.48729, size = 5466, normalized size = 26.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 2*((a*c^3)^(1/4)*a*b*c^4*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*
c^3)^(1/4)*a*b*c^4*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3
)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
```

```
abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
) - ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*c
os(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)
*a*c^3)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*arctan(-((a/c)^(1
/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*
sin(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*b
*c^5*abs(c) - (b^2 - 4*a*c)*c^6) + 2*((a*c^3)^(1/4)*a*b*c^4*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*c^4*sin(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/
4)*a*c^3)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/
4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2
*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4
)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*
c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)
*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a
*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*sin(1/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3)*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*b*c^5*abs(c) - (b^2 - 4*a*c)*c^6) - ((a*c
^3)^(1/4)*a*b*c^4*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1
```

```
/4)*a*b*c^4*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + ((a*c^3)^(3/4)*b
^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^3 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(
3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*log(-2*x*(a/c)^(1/4)*cos
(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(
b^2 - 4*a*c)*b*c^5*abs(c) - (b^2 - 4*a*c)*c^6) - ((a*c^3)^(1/4)*a*b*c^4*cos
(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*c^4*cos(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4
)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*
cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4
)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*
((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
 + 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(
(a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
- 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*
c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3 + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3)*log(-2*x*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*b*c^5*ab
```

```
s(c) - (b^2 - 4*a*c)*c^6) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

3.80 $\int \frac{x^6}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 0.0872205, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1585, 1114, 703, 634, 618, 206, 628}

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{ax+bx^3+cx^5}\,dx &= \int \frac{x^5}{a+bx^2+cx^4}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{x^2}{2c} + \frac{\operatorname{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}\\
&= \frac{x^2}{2c} - \frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} + \frac{(b^2-2ac)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2}\\
&= \frac{x^2}{2c} - \frac{b\log\left(a+bx^2+cx^4\right)}{4c^2} - \frac{(b^2-2ac)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.0484484, size = 78, normalized size = 0.96

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - b\log\left(a+bx^2+cx^4\right) + 2cx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] time = 0.003, size = 111, normalized size = 1.4

$$\frac{x^2}{2c} - \frac{b\ln\left(cx^4+bx^2+a\right)}{4c^2} - \frac{a}{c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} + \frac{b^2}{2c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/2*x^2/c-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2}{2c} - \frac{\frac{b\log\left(cx^4+bx^2+a\right)}{4c} - \frac{\left(b^2-2ac\right)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}c}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [A] time = 1.27802, size = 556, normalized size = 6.86

$$\left[\frac{2\left(b^2c-4ac^2\right)x^2-\left(b^2-2ac\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-\left(b^3-4abc\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c^2-4ac^3\right)}, \frac{2\left(b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b
*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 1.54081, size = 316, normalized size = 3.9

$$\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{4c^{\cdot}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*
log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2) + sq
```

```
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (-a*b
- 8*a*c**2*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
 - b**2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

Giac [A] time = 1.09547, size = 101, normalized size = 1.25

$$\frac{x^2}{2\,c} - \frac{b\log\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/4*b*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.81 $\int \frac{x^5}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.22733, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1585, 1122, 1166, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^5}{ax+bx^3+cx^5}\,dx = \int \frac{x^4}{a+bx^2+cx^4}\,dx$$
$$= \frac{x}{c} - \frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}$$
$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.144767, size = 202, normalized size = 1.13

$$-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.013, size = 343, normalized size = 1.9

$$\frac{x}{c} - \frac{\sqrt{2}b}{2c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} + \sqrt{2}a\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^5+b*x^3+a*x),x)
```

```
[Out] x/c-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/
2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+1/2/c*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
```

```
+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [B] time = 1.41226, size = 2168, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A] time = 1.60423, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5 - 128ab^2c^4 + 16b^4c^3\right) + t^2\left(48a^2bc^2 - 28ab^3c + 4b^5\right) + a^3, \left(t \mapsto t\log\left(x + \frac{32t^3abc^4 - 8t^3b^3c^3 - 4}{a^2c -}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

Giac [C] time = 2.33583, size = 4593, normalized size = 25.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -2*(3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(3/4)
*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*b*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^
3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2 - 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3
/4)*b*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*c^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*c^2*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1
/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*b*c^2*abs(c) - (b^2
*c - 4*a*c^2)*c^2) - 2*(3*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
)))) - (a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9
*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))) + 9*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2
```

```
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3 + (a*c^3)^(3/4)*b*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
+ (a*c^3)^(1/4)*a*c^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3
)^(1/4)*a*c^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^
(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4
)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)
*b*c^2*abs(c) - (b^2*c - 4*a*c^2)*c^2) + ((a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3
)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2 - 9*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (a*c^3)^(3/4)*b*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) - (a*c^3)^(1/4)*a*c^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*b*c^2*abs(c) - (b^2*c - 4*a*c^2)*c^
2) + ((a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*
(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*b*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*
c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*(a*c^3)^(3/4)*b*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
```

```
))))^2 - (a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 +
 3*(a*c^3)^(3/4)*b*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*
c^2*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*c^2*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi +
 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a
*c)*b*c^2*abs(c) - (b^2*c - 4*a*c^2)*c^2) + x/c
```

3.82 $\int \frac{x^4}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(4*c)
```

Rubi [A] time = 0.0673125, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1585, 1114, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(4*c)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{ax+bx^3+cx^5}\,dx &= \int \frac{x^3}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^2\right) \\
&= \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c} \\
&= \frac{\log\left(a+bx^2+cx^4\right)}{4c} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^2+cx^4\right)}{4c}
\end{aligned}$$

Mathematica [A] time = 0.0279847, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^2+cx^4\right) - \frac{2b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^2 + c*x^4])/(4*c)
```

Maple [A] time = 0.003, size = 60, normalized size = 1.

$$\frac{\ln\left(cx^4+bx^2+a\right)}{4c} - \frac{b}{2c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2/c*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{cx^5+bx^3+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^5 + b*x^3 + a*x), x)
```

Fricas [A] time = 1.31587, size = 443, normalized size = 7.03

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)},\ \frac{2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{\ldots}}{b^2-\ldots}\right)\ldots}{4\left(\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*x^2
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] time = 0.813335, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}+\frac{1}{4c}\right)\log\left(x^2+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-b*
sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c +
b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/
(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

Giac [A] time = 1.08947, size = 80, normalized size = 1.27

$$-\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c}+\frac{\log\left(cx^4+bx^2+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
4*log(c*x^4 + b*x^2 + a)/c
```

3.83 $\int \frac{x^3}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.0937336, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1585, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx = \int \frac{x^2}{a+bx^2+cx^4}\,dx$$
$$= -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.0976082, size = 165, normalized size = 1.1

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.011, size = 208, normalized size = 1.4

$$\frac{\sqrt{2}}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{\sqrt{2}b}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-1/2*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{cx^5+bx^3+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^5 + b*x^3 + a*x), x)
```

Fricas [B] time = 1.52182, size = 1206, normalized size = 8.04

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)}{\sqrt{b^2c^2-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/
2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqr
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2
*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4
*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(
b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*s
qrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a
*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A] time = 0.76821, size = 75, normalized size = 0.5

$$\operatorname{RootSum}\left(t^4\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(-16abc+4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c-2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a*
b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 2*
_t*b + x)))
```

Giac [C] time = 2.20602, size = 5350, normalized size = 35.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(a
*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arc
```

```
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (
a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2
 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)
^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2 - 4*(a
*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^3)*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) + 1/2*(3*((a*c^3)^(3/4)*b^
2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3
)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
 (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c
^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*co
s(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/
4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^
2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3
)^(3/4)*sqrt(b^2 - 4*a*c)*b)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^3)*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))
/(a*b^2*c^3 - 4*a^2*c^4) - 1/4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
(a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
```

```
c)))))^3 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(
b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^
2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4
)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3
)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*
cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
(a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(
b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*log(-2*x*(a/c)^(1/4)
*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a
*b^2*c^3 - 4*a^2*c^4) - 1/4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*
c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2
 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*s
qrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(
3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*
a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cos
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*
c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2
 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3)*log(-2*x*(a/c)^(1/4)*co
s(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^
2*c^3 - 4*a^2*c^4)
```

3.84 $\int \frac{x^2}{ax+bx^3+cx^5} dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.0426447, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1585, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{ax+bx^3+cx^5}\,dx &= \int \frac{x}{a+bx^2+cx^4}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)\\ &= -\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.0093676, size = 39, normalized size = 1.08

$$\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0., size = 36, normalized size = 1.

$$\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^5+bx^3+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^5 + b*x^3 + a*x), x)
```

Fricas [A] time = 1.53897, size = 290, normalized size = 8.06

$$\left[\frac{\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}},\,-\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] time = 0.444931, size = 131, normalized size = 3.64

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2
```

Giac [A] time = 1.11572, size = 47, normalized size = 1.31

$$\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.85 $\int \frac{x}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.079619, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1585, 1093, 205}

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{ax+bx^3+cx^5}\,dx = \int \frac{1}{a+bx^2+cx^4}\,dx$$
$$= \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.0831481, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A] time = 0.01, size = 116, normalized size = 0.8

$$-c\sqrt{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}} - c\sqrt{2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^5+b*x^3+a*x),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^5+bx^3+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^5 + b*x^3 + a*x), x)
```

Fricas [B] time = 1.57106, size = 1323, normalized size = 8.82

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A] time = 0.896009, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac-2tb^2}{c}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Giac [C] time = 1.35871, size = 1365, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 -
```

```
 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*s
qrt(a*c)*b/(a*abs(c))))))/(a*b^2*c - 4*a^2*c^2) + 1/2*(((a*c^3)^(1/4)*b^2 -
 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a
*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
)))*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))
/(a*b^2*c - 4*a^2*c^2) - 1/4*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a
*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
)) - ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*
cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*
b^2*c - 4*a^2*c^2) - 1/4*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3
)^(1/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) -
 ((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)
*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(
1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*
c - 4*a^2*c^2)
```

3.86 $\int \frac{1}{ax+bx^3+cx^5} \, dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rubi [A] time = 0.0718082, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1594, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(-1),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax+bx^3+cx^5}\,dx &= \int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right) \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} + \frac{\operatorname{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a} \\
&= \frac{\log(x)}{a} - \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a} \\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a} + \frac{b\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0732088, size = 113, normalized size = 1.64

$$\frac{-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4\log(x)\sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(-1),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.006, size = 66, normalized size = 1.

$$-\frac{\ln\left(cx^4+bx^2+a\right)}{4\,a}-\frac{b}{2\,a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^5+b*x^3+a*x),x)
```

```
[Out] -1/4*ln(c*x^4+b*x^2+a)/a-1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))+ln(x)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}}+\frac{1}{4}\,\log\left(cx^4+bx^2+a\right)}{a}+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^3 + b*x)/(c*x^4 + b*x^2 + a), x)/a + log(x)/a
```

Fricas [A] time = 1.5878, size = 510, normalized size = 7.39

$$\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(b^2-4\,ac\right)\log\left(cx^4+bx^2+a\right)+4\left(b^2-4\,ac\right)\log\left(x\right)}{4\left(ab^2-4\,a^2c\right)},\frac{2\,\sqrt{-b^2+\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] time = 2.86765, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) +
 (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
```

```
*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(b*sqr
t(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) + l
og(x)/a
```

Giac [A] time = 1.09554, size = 92, normalized size = 1.33

$$-\frac{b \arctan\left(\frac{2 cx^2+b}{\sqrt{-b^2+4 ac}}\right)}{2 \sqrt{-b^2 + 4 ac}a} - \frac{\log\left(cx^4 + bx^2 + a\right)}{4 a} + \frac{\log\left(x^2\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a
```

3.87 $\int \frac{1}{x(ax+bx^3+cx^5)}\,dx$

Optimal. Leaf size=174

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.195319, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1585, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx &= \int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx\\
&= -\frac{1}{ax}+\frac{\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx}{a}\\
&= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}\\
&= -\frac{1}{ax}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.422918, size = 191, normalized size = 1.1

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
]))/(2*a)
```

Maple [A] time = 0.019, size = 232, normalized size = 1.3

$$-\frac{c\sqrt{2}}{2a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{c\sqrt{2}b}{2a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^5+b*x^3+a*x),x)
```

```
[Out] -1/2/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*
```

```
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b-1/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

Fricas [B] time = 1.62466, size = 2279, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2
*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a
^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))
)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 -
 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5
 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^
3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
- 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*s
qrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqr
t(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*
c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x)
```

Sympy [A] time = 1.80758, size = 148, normalized size = 0.85

$$\mathrm{RootSum}\left(t^4\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2+48t^3a^4}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t**
3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c**2
+ 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

Giac [C] time = 2.35354, size = 4581, normalized size = 26.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -2*(3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(3/4)
*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*a*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^
3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2 - 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3
/4)*a*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*b*
c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*c*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1
/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a) - (b^2
*c - 4*a*c^2)*a^2) - 2*(3*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
)))) - (a*c^3)^(3/4)*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9
*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*a*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))) + 9*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2
```

```
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3 + (a*c^3)^(3/4)*a*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3
 + (a*c^3)^(1/4)*a*b*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3
)^(1/4)*a*b*c*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^
(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4
)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)
*a*b*c*abs(a) - (b^2*c - 4*a*c^2)*a^2) + ((a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3
)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2 - 9*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (a*c^3)^(3/4)*a*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*b*c*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) - (a*c^3)^(1/4)*a*b*c*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a) - (b^2*c - 4*a*c^2)*a^
2) + ((a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*
(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*a*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*
c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*(a*c^3)^(3/4)*a*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
```

```
))))^2 - (a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 +
 3*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(1/4)*a*
b*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*c*cos(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi +
 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a
*c)*a*b*c*abs(a) - (b^2*c - 4*a*c^2)*a^2) - 1/(a*x)
```

3.88 $\int \frac{1}{x^2(ax+bx^3+cx^5)}\,dx$

Optimal. Leaf size=89

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a^2
)
```

Rubi [A] time = 0.132696, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1585, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*
a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a^2
)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)}\,dx &= \int \frac{1}{x^3\left(a+bx^2+cx^4\right)}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a}\\
&= -\frac{1}{2ax^2}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)}{2a}\\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2}\\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2}+\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4a^2}\\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(b^2-2ac\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2a^2}\\
&= -\frac{1}{2ax^2}-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.124684, size = 135, normalized size = 1.52

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{x^2}-4b\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A] time = 0.006, size = 119, normalized size = 1.3

$$\frac{b\ln\left(cx^4+bx^2+a\right)}{4a^2}-\frac{c}{a}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{b^2}{2a^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))*b^2-1/2/a/x^2-b*ln(x)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\log(x)}{a^2}+\frac{\frac{1}{4}b\log\left(cx^4+bx^2+a\right)+\frac{\left(b^2-2ac\right)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}}}{a^2}-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -b*log(x)/a^2 + integrate((b*c*x^3 + (b^2 - a*c)*x)/(c*x^4 + b*x^2 + a), x)
/a^2 - 1/2/(a*x^2)
```

Fricas [A] time = 2.01744, size = 664, normalized size = 7.46

$$\left[-\frac{\left(b^2-2ac\right)\sqrt{b^2-4ac}x^2\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-\left(b^3-4abc\right)x^2\log\left(cx^4+bx^2+a\right)+4\left(b^3-4abc\right)}{4\left(a^2b^2-4a^3c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2
 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

Sympy [B] time = 7.24562, size = 345, normalized size = 3.88

$$\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2 + \frac{-8a^3c\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right) + 2a^2b^2\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right) + 3abc - b^3}{2ac^2 - b^2c}\right) + \Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*l
og(x**2 + (-8*a**3*c*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a*
*2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
- b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) + (
b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*lo
g(x**2 + (-8*a**3*c*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**
2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c -
 b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) - 1/
(2*a*x**2) - b*log(x)/a**2
```

Giac [A] time = 1.09508, size = 127, normalized size = 1.43

$$\frac{b\log\left(cx^4+bx^2+a\right)}{4\,a^2} - \frac{b\log\left(x^2\right)}{2\,a^2} + \frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2} + \frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*x
^2 - a)/(a^2*x^2)
```

3.89 $\int \frac{x^{11}}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}+\frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^4}{2c\left(b^2-4ac\right)}-\frac{b\log\left(a+b\right.}{2c}$$

```
[Out] ((b^2 - 3*a*c)*x^2)/(c^2*(b^2 - 4*a*c)) - (b*x^4)/(2*c*(b^2 - 4*a*c)) + (x^
6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c
+ 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(
3/2)) - (b*Log[a + b*x^2 + c*x^4])/(2*c^3)
```

Rubi [A] time = 0.222667, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1585, 1114, 738, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}+\frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^4}{2c\left(b^2-4ac\right)}-\frac{b\log\left(a+b\right.}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```


```
[Out] ((b^2 - 3*a*c)*x^2)/(c^2*(b^2 - 4*a*c)) - (b*x^4)/(2*c*(b^2 - 4*a*c)) + (x^
6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c
+ 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(
3/2)) - (b*Log[a + b*x^2 + c*x^4])/(2*c^3)
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^9}{\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$
$$= \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(-\frac{2\left(b^2-3ac\right)}{c^2}+\frac{2bx}{c}+\frac{2\left(a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x\right)}{c^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$
$$= \frac{\left(b^2-3ac\right)x^2}{c^2\left(b^2-4ac\right)} - \frac{bx^4}{2c\left(b^2-4ac\right)} + \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{a\left(b^2-3ac\right)+b\left(b^2-4ac\right)x}{a+bx+cx^2}\right.}{c^2\left(b^2-4ac\right)}$$
$$= \frac{\left(b^2-3ac\right)x^2}{c^2\left(b^2-4ac\right)} - \frac{bx^4}{2c\left(b^2-4ac\right)} + \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x\right.}{2c^3}$$
$$= \frac{\left(b^2-3ac\right)x^2}{c^2\left(b^2-4ac\right)} - \frac{bx^4}{2c\left(b^2-4ac\right)} + \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\log\left(a+bx^2+cx^4\right)}{2c^3} - \frac{\left(b^4\right.}{}$$
$$= \frac{\left(b^2-3ac\right)x^2}{c^2\left(b^2-4ac\right)} - \frac{bx^4}{2c\left(b^2-4ac\right)} + \frac{x^6\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-}}{c^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.20864, size = 151, normalized size = 0.91

$$\frac{-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{a^2c\left(3b-2cx^2\right)-ab^2\left(b-4cx^2\right)+b^4\left(-x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-b\log\left(a+bx^2+cx^4\right)+cx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (c*x^2 + (-(b^4*x^2) - a*b^2*(b - 4*c*x^2) + a^2*c*(3*b - 2*c*x^2))/((b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b +
 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + b*x^2 + c*x
^4])/(2*c^3)
```

Maple [B] time = 0.013, size = 383, normalized size = 2.3

$$\frac{x^2}{2c^2} + \frac{a^2x^2}{c\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)} - 2\frac{ax^2b^2}{c^2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)} + \frac{x^2b^4}{2c^3\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)} - \frac{}{2c^2\left(cx^4+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2/c^2*x^2+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2-2/c^2/(c*x^4+b*x^2+a)/(
4*a*c-b^2)*x^2*a*b^2+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^4-3/2/c^2/(c
```

```
*x^4+b*x^2+a)*a^2*b/(4*a*c-b^2)+1/2/c^3/(c*x^4+b*x^2+a)*a*b^3/(4*a*c-b^2)-2
/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*b+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a
)*b^3-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2+6/c^2
/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2-1/c^3/(4*a*c
-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ab^3 - 3\,a^2bc + \left(b^4 - 4\,ab^2c + 2\,a^2c^2\right)x^2}{2\left(ab^2c^3 - 4\,a^2c^4 + \left(b^2c^4 - 4\,ac^5\right)x^4 + \left(b^3c^3 - 4\,abc^4\right)x^2\right)} + \frac{x^2}{2\,c^2} + \frac{-2\int \frac{\left(b^3-4\,abc\right)x^3+\left(ab^2-3\,a^2c\right)x}{cx^4+bx^2+a}\,dx}{b^2c^2 - 4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b^3 - 3*a^2*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^2)/(a*b^2*c^3 - 4
*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*x^2/c
^2 + 2*integrate(-((b^3 - 4*a*b*c)*x^3 + (a*b^2 - 3*a^2*c)*x)/(c*x^4 + b*x^
2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Fricas [B] time = 1.67139, size = 1806, normalized size = 10.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^6 - a*b^5 + 7*a^2*b^3*c - 12*a
^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 - (b^6 - 9*a*b^4*c + 26
*a^2*b^2*c^2 - 24*a^3*c^3)*x^2 - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + (b^4*c
- 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 +
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^
2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b
*c^5)*x^2), 1/2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^6 - a*b^5 + 7*a^2*b
^3*c - 12*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 - (b^6 - 9*a
*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x^2 - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*
c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^
2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
4*a*c)) - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2)*log(c*x^4 + b*x^2 +
 a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*
a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

Sympy [B] time = 4.75116, size = 877, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(
2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x**2 +
 (-5*a**2*b*c - 16*a**2*c**4*(-b/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**
2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6))) + a*b**3 + 8*a*b**2*c**3*(-b/(2*c**3) - sqrt(-(4*a*c - b
**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6))) - b**4*c**2*(-b/(2*c**3) - sqrt(-(4*a*c -
 b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6))))/(6*a**2*c**2 - 6*a*b**2*c + b**4)) + (
-b/(2*c**3) + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2
*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x**2 +
(-5*a**2*b*c - 16*a**2*c**4*(-b/(2*c**3) + sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6))) + a*b**3 + 8*a*b**2*c**3*(-b/(2*c**3) + sqrt(-(4*a*c - b*
*2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6))) - b**4*c**2*(-b/(2*c**3) + sqrt(-(4*a*c -
b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6))))/(6*a**2*c**2 - 6*a*b**2*c + b**4)) + (-
3*a**2*b*c + a*b**3 + x**2*(2*a**2*c**2 - 4*a*b**2*c + b**4))/(8*a**2*c**4
- 2*a*b**2*c**3 + x**4*(8*a*c**5 - 2*b**2*c**4) + x**2*(8*a*b*c**4 - 2*b**3
*c**3)) + x**2/(2*c**2)
```

Giac [A] time = 21.8737, size = 217, normalized size = 1.31

$$\frac{\left(b^4 - 6\,ab^2c + 6\,a^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^3 - 4\,ac^4\right)\sqrt{-b^2+4\,ac}} + \frac{x^2}{2\,c^2} + \frac{b^3x^4 - 4\,abcx^4 - 2\,a^2cx^2 - a^2b}{2\left(cx^4+bx^2+a\right)\left(b^2c^2 - 4\,ac^3\right)} - \frac{b\log\left(cx^4+bx^2+a\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + 1/2*x^2/c^2 + 1/2*(b^3*x^4 - 4*a*b*c
*x^4 - 2*a^2*c*x^2 - a^2*b)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/2
*b*log(c*x^4 + b*x^2 + a)/c^3
```

3.90 $\int \frac{x^{10}}{\left(ax+bx^3+cx^5\right)^2} dx$

Optimal. Leaf size=331

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(3b^2 -\right.}{2c^2\left(b^2\right.}$$

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) +
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*
b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.702875, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1585, 1120, 1279, 1166, 205}

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(3b^2 -\right.}{2c^2\left(b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```


```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) +
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*
b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{10}}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^8}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^4\left(10a+3bx^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{x^2\left(9ab+3\left(3b^2-10ac\right)x^2\right)}{a+bx^2+cx^4}\,dx}{6c\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{3a\left(3b^2-10ac\right)+3b\left(3b^2-13ac\right)x^2 \cdots}{a+bx^2+cx^4}}{6c^2\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+\cdots}{\sqrt{b^2-4a\cdots}}\right.\cdots}{4c^2\left(b^2\cdots\right.}$$

$$= \frac{\left(3b^2-10ac\right)x}{2c^2\left(b^2-4ac\right)} - \frac{bx^3}{2c\left(b^2-4ac\right)} + \frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+\cdots}{\sqrt{b^2-4a\cdots}}\right.\cdots}{2\sqrt{2}c^{5/2}\left(b^2-4ac\cdots\right.}$$

Mathematica [A] time = 0.713003, size = 327, normalized size = 0.99

$$\frac{-\frac{\sqrt{2}\left(-20a^2c^2+3b^3\sqrt{b^2-4ac}+19ab^2c-13abc\sqrt{b^2-4ac}-3b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(20a^2c^2+3b^3\sqrt{b^2-4ac}-19ab^2c-13abc\sqrt{b^2-4ac}+3b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}\cdots}}\right.\cdots}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\cdots}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^2
```

```
+ 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[b^2 -
 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c
^(5/2))
```

Maple [B] time = 0.033, size = 844, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] x/c^2+3/2/c/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^3*a-1/2/c^2/(c*x^4+b*x^2+a)*b^3
/(4*a*c-b^2)*x^3+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x-1/2/c^2/(c*x^4+b*x^2
+a)*a/(4*a*c-b^2)*x*b^2-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+3/4/c^2/(
4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*a^2-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b
^2+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4+13/4/c/(4*
a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b-3/4/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2-19/4/c/(4*
a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2+3/4/c^2/(4*a*c-b^2)
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^{3}-3\,abc\right)x^{3}+\left(ab^{2}-2\,a^{2}c\right)x}{2\left(ab^{2}c^{2}-4\,a^{2}c^{3}+\left(b^{2}c^{3}-4\,ac^{4}\right)x^{4}+\left(b^{3}c^{2}-4\,abc^{3}\right)x^{2}\right)}+\frac{-\int\frac{3\,ab^{2}-10\,a^{2}c+\left(3\,b^{3}-13\,abc\right)x^{2}}{cx^{4}+bx^{2}+a}\,dx}{2\left(b^{2}c^{2}-4\,ac^{3}\right)}+\frac{x}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b
^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*b^
2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4
*a*c^3) + x/c^2
```

Fricas [B] time = 2.15809, size = 6311, normalized size = 19.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/2)*(a*b^2*
c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt
(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*
b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^
2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64
*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5
*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^
3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 +
336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^
6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385
*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b
^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*s
qrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051
*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*
a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818
*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6
 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c +
385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^1
3)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(
a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2
)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 -
 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 25
00*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8
818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b
^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2
)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*
x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 -
 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2
- 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 -
```

```
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10
 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^
5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(3*
a*b^2 - 10*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^
3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [A] time = 6.66133, size = 450, normalized size = 1.36

$$\frac{x^3\left(3abc-b^3\right)+x\left(2a^2c-ab^2\right)}{8a^2c^3-2ab^2c^2+x^4\left(8ac^4-2b^2c^3\right)+x^2\left(8abc^3-2b^3c^2\right)}+\operatorname{RootSum}\left(t^4\left(1048576a^6c^{11}-1572864a^5b^2c^{10}+983040a^4b^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (x**3*(3*a*b*c - b**3) + x*(2*a**2*c - a*b**2))/(8*a**2*c**3 - 2*a*b**2*c**
2 + x**4*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b**3*c**2)) + Root
Sum(_t**4*(1048576*a**6*c**11 - 1572864*a**5*b**2*c**10 + 983040*a**4*b**4*
c**9 - 327680*a**3*b**6*c**8 + 61440*a**2*b**8*c**7 - 6144*a*b**10*c**6 + 2
56*b**12*c**5) + _t**2*(430080*a**6*b*c**6 - 716800*a**5*b**3*c**5 + 483840
*a**4*b**5*c**4 - 170496*a**3*b**7*c**3 + 33232*a**2*b**9*c**2 - 3408*a*b**
11*c + 144*b**13) + 10000*a**7*c**2 - 4200*a**6*b**2*c + 441*a**5*b**4, Lam
bda(_t, _t*log(x + (65536*_t**3*a**4*b*c**9 - 61440*_t**3*a**3*b**3*c**8 +
21504*_t**3*a**2*b**5*c**7 - 3328*_t**3*a*b**7*c**6 + 192*_t**3*b**9*c**5 -
 8000*_t*a**5*c**5 + 36160*_t*a**4*b**2*c**4 - 32476*_t*a**3*b**4*c**3 + 11
592*_t*a**2*b**6*c**2 - 1836*_t*a*b**8*c + 108*_t*b**10)/(2500*a**5*c**3 -
5625*a**4*b**2*c**2 + 1971*a**3*b**4*c - 189*a**2*b**6)))) + x/c**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.91 $\int \frac{x^9}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=132

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 + c*x^4]/(4*c^2)
```

Rubi [A] time = 0.151706, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1585, 1114, 738, 773, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 + c*x^4]/(4*c^2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
```

```
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^9}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^7}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$= \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx,x,x^2\right)}{2\left(b^2-4ac\right)}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-ab+\left(-b^2+4ac\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c\left(b^2-4ac\right)}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2} - \frac{\left(b\left(b^2-6ac\right)\right)}{}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\right.}{2c^2\left(\right.}$$

$$= -\frac{bx^2}{2c\left(b^2-4ac\right)} + \frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}} + \frac{\log\left(a+bx^2\right.}{4c^2}$$

Mathematica [A] time = 0.185909, size = 121, normalized size = 0.92

$$\frac{\frac{2\left(-2a^2c+ab\left(b-3cx^2\right)+b^3x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2
 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] time = 0.011, size = 222, normalized size = 1.7

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(\frac{b\left(3\,ac-b^2\right)x^2}{c^2\left(4\,ac-b^2\right)}+\frac{a\left(2\,ac-b^2\right)}{c^2\left(4\,ac-b^2\right)}\right)+\frac{\ln\left(cx^4+bx^2+a\right)a}{c\left(4\,ac-b^2\right)}-\frac{\ln\left(cx^4+bx^2+a\right)b^2}{4\,c^2\left(4\,ac-b^2\right)}-3\,\frac{ab}{c\left(4\,ac-b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^
4+b*x^2+a)+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a-1/4/c^2/(4*a*c-b^2)*ln(c*x^4
+b*x^2+a)*b^2-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a
*b+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ab^2-2\,a^2c+\left(b^3-3\,abc\right)x^2}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^4+\left(b^3c^2-4\,abc^3\right)x^2\right)}-\frac{-\int\frac{\left(b^2-4\,ac\right)x^3+abx}{cx^4+bx^2+a}\,dx}{b^2c-4\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x^2)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c
^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) - integrate(-((b^2 - 4*a*c)*
x^3 + a*b*x)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [B] time = 1.38481, size = 1412, normalized size = 10.7

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\left(b^5-7\,ab^3c+12\,a^2bc^2\right)x^2+\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\sqrt{b^2-\cdots}}{4\left(ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c
^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*
x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2
+ b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a
^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3
*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (
b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 +
 b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

Sympy [B] time = 3.50461, size = 745, normalized size = 5.64

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)\log\left(x^2+\frac{-32a^2c^3\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)+8a^2c+16ab^2c^2 \cdots}{\cdots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(
-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c
**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(b*sqrt(-(4*a*c
 - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(b*sqrt(-(4*a*c
 - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x**2*(3*a
*b*c - b**3))/(8*a**2*c**3 - 2*a*b**2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3)
+ x**2*(8*a*b*c**3 - 2*b**3*c**2))
```

Giac [A] time = 25.6046, size = 205, normalized size = 1.55

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}}-\frac{b^2cx^4-4\,ac^2x^4-b^3x^2+2\,abcx^2-ab^2}{4\left(cx^4+bx^2+a\right)\left(b^2c^2-4\,ac^3\right)}+\frac{\log\left(cx^4+bx^2+a\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4
*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*a*
b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x^4
+ b*x^2 + a)/c^2
```

3.92 $\int \frac{x^8}{\left(ax+bx^3+cx^5\right)^2} dx$

Optimal. Leaf size=271

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-$$

```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.525544, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1585, 1120, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```


```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
```

```
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\left(ax+bx^3+cx^5\right)^2}\,dx &= \int \frac{x^6}{\left(a+bx^2+cx^4\right)^2}\,dx\\
&= \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{x^2\left(6a+bx^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}\\
&= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{ab+\left(b^2-6ac\right)x^2}{a+bx^2+cx^4}\,dx}{2c\left(b^2-4ac\right)}\\
&= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4c\left(b^2-4ac\right)}\\
&= -\frac{bx}{2c\left(b^2-4ac\right)} + \frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] time = 0.572439, size = 282, normalized size = 1.04

$$
\frac{-\frac{2\sqrt{c}x\left(a\left(b-2cx^2\right)+b^2x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+8abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}-8abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
```

```
(4*c^(3/2))
```

Maple [B] time = 0.029, size = 602, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] (-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2*a*b/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a
)+3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^2+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b-1/4/(4*a*c-b^2
)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3-3/2/(4*a*c-b^2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*a+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+2/(4*a*c-b^2)/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^2-2\,ac\right)x^3+abx}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}-\frac{-\int\frac{\left(b^2-6\,ac\right)x^2+ab}{cx^4+bx^2+a}\,dx}{2\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2
*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2 - 6*a*c)*x^2 + a*b)/
(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [B] time = 1.72308, size = 4806, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60
*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
```

```
*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*
a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 8
8*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) +
sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c
^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x
- 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c
^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((
b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 6
4*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c
^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a
*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*
a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^
4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a
*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88
*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^
6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + s
qrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^
2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x -
 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^
3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6))))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a
^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

Sympy [A] time = 4.63166, size = 379, normalized size = 1.4

$$-\frac{-abx + x^3\left(2ac - b^2\right)}{8a^2c^2 - 2ab^2c + x^4\left(8ac^3 - 2b^2c^2\right) + x^2\left(8abc^2 - 2b^3c\right)} + \mathrm{RootSum}\left(t^4\left(1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a^4b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -(-a*b*x + x**3*(2*a*c - b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3
- 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + RootSum(_t**4*(1048576*a**
6*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4*b**4*c**7 - 327680*a**3*b**6*
```

```
c**6 + 61440*a**2*b**8*c**5 - 6144*a*b**10*c**4 + 256*b**12*c**3) + _t**2*(
-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**
2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**5*c**2 - 360*a**4*b**2*c +
 25*a**3*b**4, Lambda(_t, _t*log(x + (49152*_t**3*a**4*c**7 - 40960*_t**3*a
**3*b**2*c**6 + 12288*_t**3*a**2*b**4*c**5 - 1536*_t**3*a*b**6*c**4 + 64*_t
**3*b**8*c**3 - 1728*_t*a**3*b*c**3 + 656*_t*a**2*b**3*c**2 - 88*_t*a*b**5*
c + 4*_t*b**7)/(324*a**3*c**2 - 81*a**2*b**2*c + 5*a*b**4))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.93 $\int \frac{x^7}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=78

$$\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0719037, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1585, 1114, 722, 618, 206}

$$\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(ax+bx^3+cx^5\right)^2}\,dx &= \int \frac{x^5}{\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right) \\
&= \frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{b^2-4ac} \\
&= \frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{b^2-4ac} \\
&= \frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0911856, size = 93, normalized size = 1.19

$$\frac{a\left(b-2cx^2\right)+b^2x^2}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{2a\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.009, size = 104, normalized size = 1.3

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(-\frac{\left(2\,ac-b^2\right)x^2}{c\left(4\,ac-b^2\right)}+\frac{ab}{c\left(4\,ac-b^2\right)}\right)+2\,\frac{a}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2*a/
(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.36568, size = 864, normalized size = 11.08

$$\left[-\frac{ab^3 - 4a^2bc + \left(b^4 - 6ab^2c + 8a^2c^2\right)x^2 + 2\left(ac^2x^4 + abcx^2 + a^2c\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{2\left(ab^4c - 8a^2b^2c^2 + 16a^3c^3 + \left(b^4c^2 - 8ab^2c^3 + 16a^2c^4\right)x^4 + \left(b^5c - 8ab^3c^2 + 16a^2bc^3\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^4
 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c - 8
*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a
*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*c - 8*a
^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

Sympy [B] time = 1.74624, size = 282, normalized size = 3.62

$$-a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2 + \frac{-16a^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} + 8a^2b^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} - ab^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} + ab}{2ac}\right) + a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*c
- b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a
**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)*
*3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) - (-a*b + x**2*(2*a
*c - b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**
2*(8*a*b*c**2 - 2*b**3*c))
```

Giac [A] time = 29.8736, size = 130, normalized size = 1.67

$$-\frac{2a\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^2-4ac\right)\sqrt{-b^2+4ac}} - \frac{b^2x^2 - 2acx^2 + ab}{2\left(cx^4+bx^2+a\right)\left(b^2c-4ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c
^2))
```

3.94 $\int \frac{x^6}{\left(ax+bx^3+cx^5\right)^2} \, dx$

Optimal. Leaf size=237

$$\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 +
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rubi [A] time = 0.357826, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1585, 1120, 1166, 205}

$$\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```


```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 +
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4\left(b^2-4ac\right)^{3/2}} + \frac{\left(b^2+4ac+b\sqrt{}\right.}{}$$

$$= \frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2+4ac+b\sqrt{}\right.}{2\sqrt{2}\sqrt{c}\left(b^2\right.}$$

Mathematica [A] time = 0.440714, size = 235, normalized size = 0.99

$$\frac{1}{4}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2 -
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/4
```

Maple [B] time = 0.026, size = 452, normalized size = 1.9

$$\frac{1}{cx^4+bx^2+a}\left(-\frac{bx^3}{8ac-2b^2}-\frac{ax}{4ac-b^2}\right)-\frac{\sqrt{2}b}{16ac-4b^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] (-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)-1/4/(4*a*c-b^2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-
1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+1/4/(4*a*c-b^2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c
)
```

Fricas [B] time = 1.39542, size = 3584, normalized size = 15.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*
x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b
*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b
^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*
c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
```

```
a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2
+ 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^
2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a
*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3
+ 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [A] time = 3.35899, size = 294, normalized size = 1.24

$$-\frac{2ax + bx^3}{8a^2c - 2ab^2 + x^4\left(8ac^2 - 2b^2c\right) + x^2\left(8abc - 2b^3\right)} + \operatorname{RootSum}\left(t^4\left(1048576a^6c^7 - 1572864a^5b^2c^6 + 983040a^4b^4c^5 - 32\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -(2*a*x + b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*
(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**2*
c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c**3
 - 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3
*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a
**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2*c**2
 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.95 $\int \frac{x^5}{\left(ax+bx^3+cx^5\right)^2} dx$

Optimal. Leaf size=75

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0689017, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1585, 1114, 638, 618, 206}

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```


```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(ax+bx^3+cx^5\right)^2}\,dx &= \int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)} \\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac} \\
&= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0693446, size = 79, normalized size = 1.05

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x^
2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.004, size = 77, normalized size = 1.

$$\frac{-bx^2-2\,a}{\left(8\,ac-2\,b^2\right)\left(cx^4+bx^2+a\right)} - b\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\left(4\,ac-b^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.28242, size = 778, normalized size = 10.37

$$\left[\frac{2\,ab^2-8\,a^2c+\left(b^3-4\,abc\right)x^2-\left(bcx^4+b^2x^2+ab\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\left(ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^4+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^2\right)},\ \frac{2\,ab^2-8\,a^2c+\cdots}{2\left(ab^4-8\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*
x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*x^2 +
 a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] time = 1.66677, size = 267, normalized size = 3.56

$$\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)}{2}-\frac{b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8\cdots}{\cdots}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b*
*2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a*
*2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3
) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 - (2*a + b*x**2)/(8*
a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

Giac [A] time = 25.7126, size = 111, normalized size = 1.48

$$\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}+\frac{bx^2+2\,a}{2\left(cx^4+bx^2+a\right)\left(b^2-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

3.96 $\int \frac{x^4}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(2*b -
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
+ Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.240247, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1585, 1119, 1166, 205}

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(2*b -
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
+ Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(1+\frac{2b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(b^2-4ac\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4ac}\right)\right)}{2\left(b^2-\right.}$$

$$= -\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4\right.}}{\sqrt{2}\left(b^2-4ac\right)}$$

Mathematica [A] time = 0.461759, size = 222, normalized size = 1.

$$\frac{-bx-2cx^3}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b +
 Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
 + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A] time = 0.063, size = 342, normalized size = 1.6

$$\frac{x}{8\,ac-2\,b^2}\left(x^2+\frac{1}{2\,c}\sqrt{-4\,ac+b^2}+\frac{b}{2\,c}\right)^{-1} + \frac{c\sqrt{2}b}{4\,ac-b^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2/(4*a*c-b^2)*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2/c*b)+c/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/
2/(4*a*c-b^2)*x/(x^2+1/2/c*b-1/2*(-4*a*c+b^2)^(1/2)/c)+c/(4*a*c-b^2)/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-1/2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*
c)
```

Fricas [B] time = 1.61236, size = 3623, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*
c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6
*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3
*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
+ 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256
*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a
^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
```

```
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2
- 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2
)*x - 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c
+ 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^
2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + 2*b*x)/((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [A] time = 3.48862, size = 298, normalized size = 1.35

$$\frac{bx+2cx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}+\mathrm{RootSum}\left(t^4\left(1048576a^7c^6-1572864a^6b^2c^5+983040a^5b^4c^4-327\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (b*x + 2*c*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(
8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c**6 - 1572864*a**6*b**2*c
**5 + 983040*a**5*b**4*c**4 - 327680*a**4*b**6*c**3 + 61440*a**3*b**8*c**2
- 6144*a**2*b**10*c + 256*a*b**12) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3*
b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**2*c**3 + 24*a*b**2*c**2
+ 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4 - 8192*_t**3*a**4*
b**2*c**3 + 512*_t**3*a**2*b**6*c - 64*_t**3*a*b**8 - 128*_t*a**2*b*c**2 -
16*_t*a*b**3*c - 4*_t*b**5)/(4*a*c**2 + 3*b**2*c))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.97 $\int \frac{x^3}{(ax+bx^3+cx^5)^2} dx$

Optimal. Leaf size=74

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0649478, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1585, 1107, 614, 618, 206}

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(ax+bx^3+cx^5\right)^2}\,dx &= \int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)\\
&= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{c\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac}\\
&= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(2c)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\
&= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0853882, size = 79, normalized size = 1.07

$$
-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2cx^2}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c))
```

Maple [A] time = 0.004, size = 75, normalized size = 1.

$$
\frac{2\,cx^2+b}{\left(8\,ac-2\,b^2\right)\left(cx^4+bx^2+a\right)}+2\,\frac{c}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.37787, size = 783, normalized size = 10.58

$$\left[-\frac{b^3 - 4\,abc + 2\left(b^2c - 4\,ac^2\right)x^2 + 2\left(c^2x^4 + bcx^2 + ac\right)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^4 + 2\,bcx^2 + b^2 - 2\,ac - \left(2\,cx^2 + b\right)\sqrt{b^2 - 4\,ac}}{cx^4 + bx^2 + a}\right)}{2\left(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^4 + \left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)x^2\right)}, -\frac{b^3 - 4\,abc}{2\left(ab^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*c
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 + b*c*x
^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] time = 1.57392, size = 267, normalized size = 3.61

$$-c\sqrt{-\frac{1}{\left(4ac - b^2\right)^3}}\log\left(x^2 + \frac{-16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} + 8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} - b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}} + bc}{2c^2}\right) + c\sqrt{-\frac{1}{\left(4ac - b^2\right)^3}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(-1/(4*a*c
- b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*
a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x**2)/
(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)
)
```

Giac [A] time = 31.5121, size = 111, normalized size = 1.5

$$-\frac{2\,c\arctan\left(\frac{2\,cx^2 + b}{\sqrt{-b^2 + 4\,ac}}\right)}{\left(b^2 - 4\,ac\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,cx^2 + b}{2\left(cx^4 + bx^2 + a\right)\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

3.98 $\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=252

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2}}}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.46056, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1585, 1092, 1166, 205}

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```


```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(ax+bx^3+cx^5\right)^2}\,dx &= \int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)} \\
&= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{4a\left(b^2-4ac\right)^{3/2}} + \frac{\left(c\left(b^2-1\right.\right.}{} \\
&= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2-1\right.}{2\sqrt{}}
\end{aligned}$$

Mathematica [A] time = 0.460747, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.044, size = 733, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2/c*b)*b-c/(-4*a*c+b^2
)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2/c*b)+1/4/(-4*a*c+b^
```

```
2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2/c*b)*b^2-1/4*c/(
4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^
2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2/c*b-1/2*(-4*a*c+b^2)^(1/2)/c)*b+c/(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2/c*b-1/2*(-4*a*c+b^2)^(1/2)/c)-1/4/(-4*a*c+b
^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2/c*b-1/2*(-4*a*c+b^2)^(1/2)/c)*b^2+1/4*c/
(4*a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [B] time = 1.76653, size = 4918, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6
- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 67
2*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 4
48*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*
x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
```

```
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [A] time = 4.47327, size = 394, normalized size = 1.56

$$-\frac{bcx^3 + x\left(-2ac + b^2\right)}{8a^3c - 2a^2b^2 + x^4\left(8a^2c^2 - 2ab^2c\right) + x^2\left(8a^2bc - 2ab^3\right)} + \operatorname{RootSum}\left(t^4\left(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -(b*c*x**3 + x*(-2*a*c + b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2
 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a**
9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*
c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**2*(
-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**
2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 +
 25*b**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672*_t**3
*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c + 64*_
t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_t*a**2*b
**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*c**3 + 5*
b**4*c**2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.99 $\int \frac{x}{\left(ax+bx^3+cx^5\right)^2} dx$

Optimal. Leaf size=122

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)
```

Rubi [A] time = 0.187625, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3 + c*x^5)^2,x]
```


```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$
$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax} + \frac{b(b^2-5ac)+c(b^2-4ac)x}{a(a+bx+cx^2)}\right)dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$
$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2\left(b^2-4ac\right)}$$
$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} - \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}}{4a^2\left(b^2\right.}$$
$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(b\left(b^2-6ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4a\ldots}\right.}{2a^2\left(b^2-4a\ldots\right.}$$
$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2+cx^4\right)}{4a^2}$$

Mathematica [A] time = 0.32239, size = 207, normalized size = 1.7

$$\frac{\frac{2a\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + 4\ldots}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log[
x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log
[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c -
b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [B] time = 0.014, size = 253, normalized size = 2.1

$$-\frac{bcx^2}{2\,a\left(cx^4+bx^2+a\right)\left(4\,ac-b^2\right)} + \frac{c}{\left(cx^4+bx^2+a\right)\left(4\,ac-b^2\right)} - \frac{b^2}{2\,a\left(cx^4+bx^2+a\right)\left(4\,ac-b^2\right)} - \frac{c\ln\left(cx^4+bx^2+a\right)}{a\left(4\,ac-b^2\right)} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x^2+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c-
1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2-1/a/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)+1
/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2))*b*c+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*b^3+ln(x)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx^2 + b^2 - 2\,ac}{2\left(\left(ab^2c - 4\,a^2c^2\right)x^4 + a^2b^2 - 4\,a^3c + \left(ab^3 - 4\,a^2bc\right)x^2\right)} + \frac{-\int \frac{\left(b^2c - 4\,ac^2\right)x^3 + \left(b^3 - 5\,abc\right)x}{cx^4 + bx^2 + a}\,dx}{a^2b^2 - 4\,a^3c} + \frac{\log\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
+ (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-((b^2*c - 4*a*c^2)*x^3 + (b^3 - 5*a
*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + log(x)/a^2
```

Fricas [B] time = 1.78416, size = 1728, normalized size = 14.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 +
 ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^
2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^
4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a*b^4 - 12
*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b
*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*
a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
]
```

Sympy [B] time = 40.9679, size = 772, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2))*log(x**2 + (-32*a**4*c**2*(
-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) + 16*a**3*b**2*c*(-b*sqrt(-(
4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 2*a**2*b**4*(-b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) - 1/(4*a**2)) - 8*a**2*c**2 + 7*a*b**2*c - b**4)/(6*a*b*c**2 - b*
*3*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 -
 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2))*log(x**2 + (-32*a**
4*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) + 16*a**3*b**2*c*(b*s
qrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 2*a**2*b**4*(b*sqrt(-(4*a*c -
b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - 1/(4*a**2)) - 8*a**2*c**2 + 7*a*b**2*c - b**4)/(6*a*b*c**2
 - b**3*c)) - (-2*a*c + b**2 + b*c*x**2)/(8*a**3*c - 2*a**2*b**2 + x**4*(8*
a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + log(x)/a**2
```

Giac [A] time = 22.1448, size = 224, normalized size = 1.84

$$-\frac{\left(b^3-6\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}+\frac{b^2cx^4-4\,ac^2x^4+b^3x^2-2\,abcx^2+3\,ab^2-8\,a^2c}{4\left(cx^4+bx^2+a\right)\left(a^2b^2-4\,a^3c\right)}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4
*a^3*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*a*
b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) - 1/
4*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

3.100 $\int \frac{1}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{3b^2-10ac}{2a^2x\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-\cdots\right.}{2\sqrt{2}a^2\left(b^2-4ac\right)\cdots}$$

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b
^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 -
10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.35189, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1594, 1121, 1281, 1166, 205}

$$-\frac{3b^2-10ac}{2a^2x\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-\cdots\right.}{2\sqrt{2}a^2\left(b^2-4ac\right)\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(-2),x]
```

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b
^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 -
10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3b^2+10ac-3bcx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-b\left(3b^2-13ac\right)-c\left(3b^2-10ac\right)x^2}{a+bx^2+cx^4}\,dx}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\right)\int}{4a^2\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\sqrt{c}\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-4ac}}-\frac{16abc}{\sqrt{b^2-4ac}}\right)\tan}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2}}}$$

Mathematica [A] time = 0.668442, size = 302, normalized size = 0.98

$$\frac{-\frac{2x\left(-3abc-2ac^2x^2+b^2cx^2+b^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)t}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(-2),x]
```

```
[Out] (-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4
*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2
]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4
```

```
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [B] time = 0.03, size = 712, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b
^2)*x^3*b^2-3/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x+1/2/a^2/(c*x^4+b*x^2+a)
*b^3/(4*a*c-b^2)*x-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+3/4/a^2*c/(4*a*
c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b^3+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-3/4/a^2*c/(4*a*c-b^2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3
-1/a^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(3\,b^2c-10\,ac^2\right)x^4+2\,ab^2-8\,a^2c+\left(3\,b^3-11\,abc\right)x^2}{2\left(\left(a^2b^2c-4\,a^3c^2\right)x^5+\left(a^2b^3-4\,a^3bc\right)x^3+\left(a^3b^2-4\,a^4c\right)x\right)}+\frac{-\int\frac{3\,b^3-13\,abc+\left(3\,b^2c-10\,ac^2\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(a^2b^2-4\,a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^2
)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a
^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/(c*
x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Fricas [B] time = 2.10783, size = 6460, normalized size = 20.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c
)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3
+ (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
+ 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^
5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*
a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*
b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^
5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7
*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-
(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sq
rt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 144
08*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*
c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/
2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*
a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b
^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*
c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7
*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^10
 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1
280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 -
4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a
^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 -
1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sqrt(1/2)*(27*b^1
1 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
- 5200*a^5*b*c^5 + (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*
b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3
```

```
*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2
)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [A] time = 7.14549, size = 481, normalized size = 1.56

$$\text{RootSum}\left(t^4\left(1048576a^{11}c^6 - 1572864a^{10}b^2c^5 + 983040a^9b^4c^4 - 327680a^8b^6c^3 + 61440a^7b^8c^2 - 6144a^6b^{10}c + 256a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] RootSum(_t**4*(1048576*a**11*c**6 - 1572864*a**10*b**2*c**5 + 983040*a**9*b
**4*c**4 - 327680*a**8*b**6*c**3 + 61440*a**7*b**8*c**2 - 6144*a**6*b**10*c
 + 256*a**5*b**12) + _t**2*(430080*a**6*b*c**6 - 716800*a**5*b**3*c**5 + 48
3840*a**4*b**5*c**4 - 170496*a**3*b**7*c**3 + 33232*a**2*b**9*c**2 - 3408*a
*b**11*c + 144*b**13) + 10000*a**2*c**7 - 4200*a*b**2*c**6 + 441*b**4*c**5,
 Lambda(_t, _t*log(x + (-81920*_t**3*a**10*c**5 + 139264*_t**3*a**9*b**2*c*
*4 - 86016*_t**3*a**8*b**4*c**3 + 25088*_t**3*a**7*b**6*c**2 - 3520*_t**3*a
**6*b**8*c + 192*_t**3*a**5*b**10 - 27200*_t*a**5*b*c**5 + 60176*_t*a**4*b*
*3*c**4 - 42448*_t*a**3*b**5*c**3 + 13320*_t*a**2*b**7*c**2 - 1944*_t*a*b**
9*c + 108*_t*b**11)/(2500*a**3*c**6 - 5625*a**2*b**2*c**5 + 1971*a*b**4*c**
4 - 189*b**6*c**3)))) - (8*a**2*c - 2*a*b**2 + x**4*(10*a*c**2 - 3*b**2*c)
+ x**2*(11*a*b*c - 3*b**3))/(x**5*(8*a**3*c**2 - 2*a**2*b**2*c) + x**3*(8*a
**3*b*c - 2*a**2*b**3) + x*(8*a**4*c - 2*a**3*b**2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.101 $\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=162

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}+\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+bcx}{2ax^2\left(b^2-4ac\right)\left(a+bx\right.}$$

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^
2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x]
)/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)
```

Rubi [A] time = 0.250251, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}+\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}+\frac{-2ac+b^2+bcx}{2ax^2\left(b^2-4ac\right)\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```


```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^
2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x]
)/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)
```


Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$
$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2\left(b^2-3ac\right)-2bcx}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2a\left(b^2-4ac\right)}$$
$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{2\left(-b^2+3ac\right)}{ax^2} - \frac{2b\left(-b^2+4ac\right)}{a^2x} + \frac{2\left(-b^4+5ab^2c-3a^2c^2-bc\left(b^2-\right.\right.}{a^2\left(a+bx+cx^2\right)}\right.\right.}{2a\left(b^2-4ac\right)}$$
$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} - \frac{\text{Subst}\left(\int \frac{-b^4+5ab^2c-3a^2c^2-}{a+bx+c}\right.}{a^3\left(b^2-\right.}$$
$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x\right.}{2a^3}$$
$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx^2+cx^4\right)}{2a^3} - \cdots$$
$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right.}{a^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.274994, size = 248, normalized size = 1.53

$$\frac{\frac{\left(6a^2c^2+b^3\sqrt{b^2-4ac}-6ab^2c-4abc\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{\left(-6a^2c^2+b^3\sqrt{b^2-4ac}+6ab^2c-4abc\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} - \frac{a\left(-3abc-\right.}{\left(b^2-4a\right.}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] (-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 -
4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
b^2 - 4*a*c)^(3/2))/(2*a^3)
```

Maple [B] time = 0.017, size = 352, normalized size = 2.2

$$-\frac{c^2x^2}{a\left(cx^4+bx^2+a\right)\left(4\,ac-b^2\right)} + \frac{cx^2b^2}{2\,a^2\left(cx^4+bx^2+a\right)\left(4\,ac-b^2\right)} - \frac{3\,bc}{2\,a\left(cx^4+bx^2+a\right)\left(4\,ac-b^2\right)} + \frac{b^3}{2\,a^2\left(cx^4+bx^2+a\right)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b
^2)*x^2*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*c+1/2/a^2/(c*x^4+b*x^2+a)*b
^3/(4*a*c-b^2)+2/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*b-1/2/a^3/(4*a*c-b^2)*
ln(c*x^4+b*x^2+a)*b^3-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^
(1/2))*c^2+6/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^
2*c-1/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4-1/2/a
^2/x^2-2*b*ln(x)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(b^2c-3\,ac^2\right)x^4+ab^2-4\,a^2c+\left(2\,b^3-7\,abc\right)x^2}{2\left(\left(a^2b^2c-4\,a^3c^2\right)x^6+\left(a^2b^3-4\,a^3bc\right)x^4+\left(a^3b^2-4\,a^4c\right)x^2\right)}-\frac{-2\int\frac{\left(b^3c-4\,abc^2\right)x^3+\left(b^4-5\,ab^2c+3\,a^2c^2\right)x}{cx^4+bx^2+a}\,dx}{a^3b^2-4\,a^4c}-\frac{2\,b\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*x^2)/((
a^2*b^2*c - 4*a^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c
)*x^2) - 2*integrate(-((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 5*a*b^2*c + 3*a^2*c
^2)*x)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c) - 2*b*log(x)/a^3
```

Fricas [B] time = 2.19399, size = 2103, normalized size = 12.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12
*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6*a
*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 -
6*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)
*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4
*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 +
 (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^
2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*
b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2
+ a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*
b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b
*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [B] time = 72.9524, size = 906, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (b/(2*a**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2
*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x**2 +
(-16*a**5*c**2*(b/(2*a**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b*
*2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
))) + 8*a**4*b**2*c*(b/(2*a**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6
*a*b**2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6))) - a**3*b**4*(b/(2*a**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
6*a*b**2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6))) + 11*a**2*b*c**2 - 7*a*b**3*c + b**5)/(6*a**2*c**3 - 6*a*b**2*c**
2 + b**4*c)) + (b/(2*a**3) + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b*
*2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
)))*log(x**2 + (-16*a**5*c**2*(b/(2*a**3) + sqrt(-(4*a*c - b**2)**3)*(6*a**
2*c**2 - 6*a*b**2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6))) + 8*a**4*b**2*c*(b/(2*a**3) + sqrt(-(4*a*c - b**2)**3)*(
6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6))) - a**3*b**4*(b/(2*a**3) + sqrt(-(4*a*c - b**2)**3)*
(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6))) + 11*a**2*b*c**2 - 7*a*b**3*c + b**5)/(6*a**2*c**3
 - 6*a*b**2*c**2 + b**4*c)) - (4*a**2*c - a*b**2 + x**4*(6*a*c**2 - 2*b**2*
c) + x**2*(7*a*b*c - 2*b**3))/(x**6*(8*a**3*c**2 - 2*a**2*b**2*c) + x**4*(8
*a**3*b*c - 2*a**2*b**3) + x**2*(8*a**4*c - 2*a**3*b**2)) - 2*b*log(x)/a**3
```

Giac [A] time = 25.5326, size = 246, normalized size = 1.52

$$\frac{\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}}-\frac{2\,b^2cx^4-6\,ac^2x^4+2\,b^3x^2-7\,abcx^2+ab^2-4\,a^2c}{2\left(cx^6+bx^4+ax^2\right)\left(a^2b^2-4\,a^3c\right)}+\frac{b\log\left(cx^4+bx^2+a\right)}{2\,a^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2*b
^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 -
 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3
```

3.102 $\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{\ldots}\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\ldots}$$

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*x^3) + (b*(5*b^2 - 19*a*c))/(2*a^3*(
b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2
 + c*x^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c
]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^
2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 3.07713, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1585, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{\ldots}\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*x^3) + (b*(5*b^2 - 19*a*c))/(2*a^3*(
b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2
 + c*x^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c
]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^
2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
```

```
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)^2}\,dx=\int \frac{1}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

$$=\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-5b^2+14ac-5bcx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$=-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}+\frac{\int \frac{-3b\left(5b^2-19ac\right)-3c\left(5b^2-14ac\right)x^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{6a^2\left(b^2-4ac\right)}$$

$$=-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3}+\frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-3\left(5b^4-24ab^2c+\cdots\right.}{a\cdots}}{6a^3\cdots}$$

$$=-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3}+\frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}-\frac{\left(c\left(5b^4-29ab^2\cdots\right.\right.}{\cdots}$$

$$=-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3}+\frac{b\left(5b^2-19ac\right)}{2a^3\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(5b^4-29ab\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.788114, size = 344, normalized size = 0.95

$$\frac{\frac{6x\left(2a^2c^2-4ab^2c-3abc^2x^2+b^3cx^2+b^4\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2+5b^3\sqrt{b^2-4ac}-29ab^2c-19abc\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-28a^2c^2+5b^3\sqrt{b^2-4ac}+29\cdots\right.}{\left(b^2-4\cdots\right.}}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] ((-4*a)/x^3 + (24*b)/x + (6*x*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*
a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b
^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29
*a*b^2*c - 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [B] time = 0.036, size = 913, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 3/2/a^2/(c*x^4+b*x^2+a)*b*c^2/(4*a*c-b^2)*x^3-1/2/a^3/(c*x^4+b*x^2+a)*b^3*c
/(4*a*c-b^2)*x^3-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c^2+2/a^2/(c*x^4+b*x^2+a
)/(4*a*c-b^2)*x*b^2*c-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^4+19/4/a^2*c^
2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-5/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^3+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-29/4/a^2*c^2
/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+5/4/a^3*c/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4-19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b+5/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3+7/a*c^3/(
4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-29/4/a^2*c^2/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+5/4/a^3*c/(4*a*c-b^2)/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4-1/3/a^2/x^3+2/a^3*b/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(5\,b^3c-19\,abc^2\right)x^6+\left(15\,b^4-62\,ab^2c+14\,a^2c^2\right)x^4-2\,a^2b^2+8\,a^3c+10\left(ab^3-4\,a^2bc\right)x^2}{6\left(\left(a^3b^2c-4\,a^4c^2\right)x^7+\left(a^3b^3-4\,a^4bc\right)x^5+\left(a^4b^2-4\,a^5c\right)x^3\right)}-\frac{-\int\frac{5\,b^4-24\,ab^2c+14\,a^2c^2+(5\,b}{cx^4+bx^2+a}}{2\left(a^3b^2-4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*x^6 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x^4
- 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2
)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) - 1/2*integrat
```

```
e(-(5*b^4 - 24*a*b^2*c + 14*a^2*c^2 + (5*b^3*c - 19*a*b*c^2)*x^2)/(c*x^4 +
b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Fricas [B] time = 2.94168, size = 7960, normalized size = 22.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*x^6 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x
^4 - 4*a^2*b^2 + 16*a^3*c + 20*(a*b^3 - 4*a^2*b*c)*x^2 + 3*sqrt(1/2)*((a^3*
b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^
3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260
*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((
625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^
4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 504
21*a^3*b^2*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c
+ 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*
b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c +
700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*
sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76
686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4
*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^
2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 4
8*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8
*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6
*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6
 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c
 - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sq
rt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*
b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b
^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^1
6*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^
3*b^2*c^7 + 9604*a^4*c^8)*x - 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 189
40*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c
^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c + 700*a
^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt(
(625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a
^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c +
48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5
*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2
- 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12
*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*
a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(
25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4
 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 -
 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4
- 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2
*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^
```

```
3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2
*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 18940*a^
2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 +
79408*a^6*b^2*c^6 - 10976*a^7*c^7 + (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^
7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2
- 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*
c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 836
30*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^1
4*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*
b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c
^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(25*b^
9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a
^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250
*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 241
08*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2
- 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*l
og((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7
+ 9604*a^4*c^8)*x - 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 18940*a^2*b^1
0*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408
*a^6*b^2*c^6 - 10976*a^7*c^7 + (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2
 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*b^12
- 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4
 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^
2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 241
5*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^
3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6
 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c
 + 48*a^9*b^2*c^2 - 64*a^10*c^3))))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3
 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)
```

Sympy [A] time = 12.0217, size = 566, normalized size = 1.57

$$\mathrm{RootSum}\left(t^4\left(1048576a^{13}c^6 - 1572864a^{12}b^2c^5 + 983040a^{11}b^4c^4 - 327680a^{10}b^6c^3 + 61440a^9b^8c^2 - 6144a^8b^{10}c + 256\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] RootSum(_t**4*(1048576*a**13*c**6 - 1572864*a**12*b**2*c**5 + 983040*a**11*
b**4*c**4 - 327680*a**10*b**6*c**3 + 61440*a**9*b**8*c**2 - 6144*a**8*b**10
*c + 256*a**7*b**12) + _t**2*(-1290240*a**7*b*c**7 + 3440640*a**6*b**3*c**6
 - 3515904*a**5*b**5*c**5 + 1870848*a**4*b**7*c**4 - 572272*a**3*b**9*c**3
+ 101856*a**2*b**11*c**2 - 9840*a*b**13*c + 400*b**15) + 38416*a**2*c**9 -
17640*a*b**2*c**8 + 2025*b**4*c**7, Lambda(_t, _t*log(x + (-212992*_t**3*a*
*12*b*c**5 + 299008*_t**3*a**11*b**3*c**4 - 164864*_t**3*a**10*b**5*c**3 +
44800*_t**3*a**9*b**7*c**2 - 6016*_t**3*a**8*b**9*c + 320*_t**3*a**7*b**11
- 21952*_t*a**7*c**7 + 289856*_t*a**6*b**2*c**6 - 682820*_t*a**5*b**4*c**5
+ 642828*_t*a**4*b**6*c**4 - 302316*_t*a**3*b**8*c**3 + 75760*_t*a**2*b**10
*c**2 - 9700*_t*a*b**12*c + 500*_t*b**14)/(9604*a**4*c**8 - 50421*a**3*b**2
*c**7 + 43410*a**2*b**4*c**6 - 12325*a*b**6*c**5 + 1125*b**8*c**4)))) + (-8
*a**3*c + 2*a**2*b**2 + x**6*(57*a*b*c**2 - 15*b**3*c) + x**4*(-14*a**2*c**
2 + 62*a*b**2*c - 15*b**4) + x**2*(40*a**2*b*c - 10*a*b**3))/(x**7*(24*a**4
```

```
*c**2 - 6*a**3*b**2*c) + x**5*(24*a**4*b*c - 6*a**3*b**3) + x**3*(24*a**5*c
 - 6*a**4*b**2))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.103 $\int \frac{1}{x^3(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=219

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{3/2}}+\frac{b\left(3b^2-11ac\right)}{2a^3x^2\left(b^2-4ac\right)}-\frac{3b^2-8ac}{4a^2x^4\left(b^2-4ac\right)}-\frac{\left(3b^2-2ac\right)\log\left(a+bx^2+cx^4\right)}{4a^4}$$

```
[Out] -(3*b^2 - 8*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(3*b^2 - 11*a*c))/(2*a^3*(b
^2 - 4*a*c)*x^2) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^4*(a + b*x^
2 + c*x^4)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^
4 - ((3*b^2 - 2*a*c)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rubi [A] time = 0.312461, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{3/2}}+\frac{b\left(3b^2-11ac\right)}{2a^3x^2\left(b^2-4ac\right)}-\frac{3b^2-8ac}{4a^2x^4\left(b^2-4ac\right)}-\frac{\left(3b^2-2ac\right)\log\left(a+bx^2+cx^4\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -(3*b^2 - 8*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(3*b^2 - 11*a*c))/(2*a^3*(b
^2 - 4*a*c)*x^2) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^4*(a + b*x^
2 + c*x^4)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^
4 - ((3*b^2 - 2*a*c)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
```

```
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3(ax+bx^3+cx^5)^2}\,dx = \int \frac{1}{x^5(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx+cx^2)^2}\,dx, x, x^2\right)$$
$$= \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^4(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{-3b^2+8ac-3bcx}{x^3(a+bx+cx^2)}\,dx, x, x^2\right)}{2a(b^2-4ac)}$$
$$= \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^4(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int\left(\frac{-3b^2+8ac}{ax^3} + \frac{3b^3-11abc}{a^2x^2} + \frac{(b^2-4ac)(-3b^2+2ac)}{a^3x} + \cdots\right.\right.}{2a(b^2-4\cdots}$$
$$= -\frac{3b^2-8ac}{4a^2(b^2-4ac)x^4} + \frac{b(3b^2-11ac)}{2a^3(b^2-4ac)x^2} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^4(a+bx^2+cx^4)} + \frac{(3b^2-2ac)\cdots}{a^4}$$
$$= -\frac{3b^2-8ac}{4a^2(b^2-4ac)x^4} + \frac{b(3b^2-11ac)}{2a^3(b^2-4ac)x^2} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^4(a+bx^2+cx^4)} + \frac{(3b^2-2ac)\cdots}{a^4}$$
$$= -\frac{3b^2-8ac}{4a^2(b^2-4ac)x^4} + \frac{b(3b^2-11ac)}{2a^3(b^2-4ac)x^2} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^4(a+bx^2+cx^4)} + \frac{(3b^2-2ac)\cdots}{a^4}$$
$$= -\frac{3b^2-8ac}{4a^2(b^2-4ac)x^4} + \frac{b(3b^2-11ac)}{2a^3(b^2-4ac)x^2} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^4(a+bx^2+cx^4)} + \frac{b(3b^4-20\cdots}{\cdots}$$

Mathematica [A] time = 0.365738, size = 328, normalized size = 1.5

$$\frac{\frac{2a(2a^2c^2-4ab^2c-3abc^2x^2+b^3cx^2+b^4)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{(8a^2c^2\sqrt{b^2-4ac}+30a^2bc^2+3b^4\sqrt{b^2-4ac}-20ab^3c-14ab^2c\sqrt{b^2-4ac}+3b^5)\log(-\sqrt{b^2-4ac}+b+2cx^2)}{(b^2-4ac)^{3/2}} + \frac{(-8a^2c^2\sqrt{b^2-4\cdots}}{\cdots}}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] (-(a^2/x^4) + (4*a*b)/x^2 + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 -
 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(3*b^2 - 2*a*c)*Lo
g[x] - ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2 + 3*b^4*Sqrt[b^2 - 4*a*c] - 14*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 8*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 -
4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^
2 - 3*b^4*Sqrt[b^2 - 4*a*c] + 14*a*b^2*c*Sqrt[b^2 - 4*a*c] - 8*a^2*c^2*Sqrt
[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(
4*a^4)
```

Maple [B] time = 0.019, size = 443, normalized size = 2.

$$\frac{3c^2bx^2}{2a^2(cx^4+bx^2+a)(4ac-b^2)} - \frac{b^3cx^2}{2a^3(cx^4+bx^2+a)(4ac-b^2)} - \frac{c^2}{a(cx^4+bx^2+a)(4ac-b^2)} + 2\frac{b^2c}{a^2(cx^4+bx^2+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 3/2/a^2/(c*x^4+b*x^2+a)*b*c^2/(4*a*c-b^2)*x^2-1/2/a^3/(c*x^4+b*x^2+a)*b^3*c
/(4*a*c-b^2)*x^2-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c^2+2/a^2/(c*x^4+b*x^2+a)/
(4*a*c-b^2)*b^2*c-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^4+2/a^2/(4*a*c-b^2)
*c^2*ln(c*x^4+b*x^2+a)-7/2/a^3/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*b^2+3/4/a^4/
(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^4+15/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+
b)/(4*a*c-b^2)^(1/2))*b*c^2-10/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*b^3*c+3/2/a^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))*b^5-1/4/a^2/x^4-2/a^3*ln(x)*c+3/a^4*ln(x)*b^2+1/a^3*b/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(3\,b^3c-11\,abc^2\right)x^6+\left(6\,b^4-25\,ab^2c+8\,a^2c^2\right)x^4-a^2b^2+4\,a^3c+3\left(ab^3-4\,a^2bc\right)x^2}{4\left(\left(a^3b^2c-4\,a^4c^2\right)x^8+\left(a^3b^3-4\,a^4bc\right)x^6+\left(a^4b^2-4\,a^5c\right)x^4\right)}-\frac{\frac{1}{4}\left(3\,b^4-14\,ab^2c+8\,a^2c^2\right)\log\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*b^3*c - 11*a*b*c^2)*x^6 + (6*b^4 - 25*a*b^2*c + 8*a^2*c^2)*x^4 -
a^2*b^2 + 4*a^3*c + 3*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2)*x^8
 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)*x^4) - integrate(((3*b^4
*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^3 + (3*b^5 - 17*a*b^3*c + 19*a^2*b*c^2)*x)
/(c*x^4 + b*x^2 + a), x)/(a^4*b^2 - 4*a^5*c) + (3*b^2 - 2*a*c)*log(x)/a^4
```

Fricas [B] time = 2.7237, size = 2627, normalized size = 12.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 + ((3*b^5*c - 20*a*b^3
*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^6 + (3*a
*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^
7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4
*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*((3*b^6*c
 - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c +
64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c
^2 - 32*a^4*c^3)*x^4)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8
 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16
*a^7*c^2)*x^4), -1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 2
3*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c
^2 - 32*a^4*c^3)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 - 2*((3
*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^
2*c^2)*x^6 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(-b^2 + 4*a*c
```

```
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((3*b^6*c - 26*
a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2
*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 3
2*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*((3*b^6*c - 26*a*b^4*c^2 + 64*a^
2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3
*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*l
og(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3
*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4)]
```

Sympy [B] time = 131.412, size = 1074, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(4*a**4*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)
/(4*a**4))*log(x**2 + (32*a**6*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c
**2 - 20*a*b**2*c + 3*b**4)/(4*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(4*a**4)) - 16*a**5*b**2*c*(-b*sqrt(-(
4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(4*a**4*(64*a**3*c*
*3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(4*a**4))
+ 2*a**4*b**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*
b**4)/(4*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2
*a*c - 3*b**2)/(4*a**4)) - 16*a**3*c**3 + 47*a**2*b**2*c**2 - 23*a*b**4*c +
 3*b**6)/(30*a**2*b*c**3 - 20*a*b**3*c**2 + 3*b**5*c)) + (b*sqrt(-(4*a*c -
b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(4*a**4*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(4*a**4))*log(x**2
 + (32*a**6*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c +
3*b**4)/(4*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
(2*a*c - 3*b**2)/(4*a**4)) - 16*a**5*b**2*c*(b*sqrt(-(4*a*c - b**2)**3)*(30
*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(4*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**
2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(4*a**4)) + 2*a**4*b**4*(b*sqrt
(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(4*a**4*(64*a**3
*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(4*a**4
)) - 16*a**3*c**3 + 47*a**2*b**2*c**2 - 23*a*b**4*c + 3*b**6)/(30*a**2*b*c*
*3 - 20*a*b**3*c**2 + 3*b**5*c)) + (-4*a**3*c + a**2*b**2 + x**6*(22*a*b*c*
*2 - 6*b**3*c) + x**4*(-8*a**2*c**2 + 25*a*b**2*c - 6*b**4) + x**2*(12*a**2
*b*c - 3*a*b**3))/(x**8*(16*a**4*c**2 - 4*a**3*b**2*c) + x**6*(16*a**4*b*c
- 4*a**3*b**3) + x**4*(16*a**5*c - 4*a**4*b**2)) - (2*a*c - 3*b**2)*log(x)/
a**4
```

Giac [A] time = 23.7638, size = 370, normalized size = 1.69

$$-\frac{\left(3\,b^{5}-20\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(a^{4}b^{2}-4\,a^{5}c\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,b^{4}cx^{4}-14\,ab^{2}c^{2}x^{4}+8\,a^{2}c^{3}x^{4}+3\,b^{5}x^{2}-12\,ab^{3}cx^{2}+2\,a^{2}bc^{2}x^{2}+}{4\,\left(a^{4}b^{2}-4\,a^{5}c\right)\left(cx^{4}+bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4
*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*x^4 - 14*a*b
^2*c^2*x^4 + 8*a^2*c^3*x^4 + 3*b^5*x^2 - 12*a*b^3*c*x^2 + 2*a^2*b*c^2*x^2 +
 5*a*b^4 - 22*a^2*b^2*c + 12*a^3*c^2)/((a^4*b^2 - 4*a^5*c)*(c*x^4 + b*x^2 +
 a)) - 1/4*(3*b^2 - 2*a*c)*log(c*x^4 + b*x^2 + a)/a^4 + 1/2*(3*b^2 - 2*a*c)
*log(x^2)/a^4 - 1/4*(9*b^2*x^4 - 6*a*c*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

3.104 $\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=142

$$\frac{2x^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (2*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^5]
)
```

Rubi [A] time = 0.170831, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1930, 1141, 510}

$$\frac{2x^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^5]
)
```

Rule 1930

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Dist[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n
 - q) + c*x^(2*(n - q)))^p), Int[x^(m + p*q)*(a + b*x^(n - q) + c*x^(2*(n -
 q)))^p, x], x] /; FreeQ[{a, b, c, m, n, p, q}, x] && EqQ[r, 2*n - q] &&  !
IntegerQ[p] && PosQ[n - q]
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}$$
$$= \frac{\left(\sqrt{x}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{x}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{ax+bx^3+cx^5}}$$
$$= \frac{2x^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

Mathematica [A] time = 0.0951912, size = 170, normalized size = 1.2

$$\frac{2x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{3\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1
/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])])/(3*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.105 $\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=380

$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}-\frac{2x^{3/2}\left(b^2-3ac\right)(a}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{}}$$

```
[Out] (-2*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(15*c^(3/2)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b + 3*c*x^2)*Sqrt[a*x + b*x^
3 + c*x^5])/(15*c) + (2*a^(1/4)*(b^2 - 3*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a*x + b
*x^3 + c*x^5]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(7
/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.287345, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1919, 1953, 1197, 1103, 1195}

$$-\frac{2x^{3/2}\left(b^2-3ac\right)\left(a+bx^2+cx^4\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (-2*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(15*c^(3/2)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b + 3*c*x^2)*Sqrt[a*x + b*x^
3 + c*x^5])/(15*c) + (2*a^(1/4)*(b^2 - 3*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a*x + b
*x^3 + c*x^5]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(7
/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx = \frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{\int \frac{\sqrt{x}\left(-ab-2\left(b^2-3ac\right)x^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx}{15c}$$

$$= \frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{-ab-2\left(b^2-3ac\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15c\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{\left(2\sqrt{a}\left(b^2-3ac\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^{3/2}\sqrt{ax+bx^3+cx^5}} +$$

$$= -\frac{2\left(b^2-3ac\right)x^{3/2}\left(a+bx^2+cx^4\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{2\sqrt[4]{a}\left(b^2-3a\right.}{}$$

Mathematica [C] time = 1.51973, size = 486, normalized size = 1.28

$$\sqrt{x}\left(i\left(b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}+4abc-b^3\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4a}}}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a + b*x^2 +
c*x^4) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*
c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(
-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b
^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])]))/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*
x^4)])
```

Maple [B] time = 0.069, size = 1042, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] -1/30*(x*(c*x^4+b*x^2+a))^(1/2)*(-6*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^7
*b*c^2-6*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^7*c^2-8*(
1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^5*b^2*c-8*(1/a*(-b+(-4*a*c+b^2)^(1/2))
)^(1/2)*(-4*a*c+b^2)^(1/2)*x^5*b*c-6*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^
3*a*b*c-6*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*a*c-2*
(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*b^3-2*(1/a*(-b+(-4*a*c+b^2)^(1/2)))
^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b^2+12*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)
/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1
/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-
2*a*c+b^2)/a/c)^(1/2))*a^2*c-3*b^2*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)
/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1
/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-
2*a*c+b^2)/a/c)^(1/2))+b*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*
((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(
-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)
/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)-12*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a
)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2
)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*
a*c+b^2)/a/c)^(1/2))*a^2*c+4*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2
)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*(1/a
*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^
2)/a/c)^(1/2))*a*b^2-2*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*a*b^2-2*(1/a*(
-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x*a*b)/x^(1/2)/(c*x^4+b*x^
2+a)/c/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^5 + bx^3 + ax}x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^5 + bx^3 + ax}x^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} \sqrt{x \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^5 + bx^3 + ax}x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

3.106 $\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=129

$$\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}\,(b^2-4ac)\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] ((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) - ((b^2 - 4*a*c)*Sq
rt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/(16*c^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.0933199, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1918, 1914, 1107, 621, 206}

$$\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}\,(b^2-4ac)\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```


```
[Out] ((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) - ((b^2 - 4*a*c)*Sq
rt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/(16*c^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
```


Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx &= \frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{\left(b^2-4ac\right)\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{8c} \\
&= \frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{\left(\left(b^2-4ac\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx}{8c\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{\left(\left(b^2-4ac\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x}{16c\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{\left(\left(b^2-4ac\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2}{\sqrt{a+bx}}}{8c\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{\left(b^2-4ac\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}
\end{aligned}
$$

Mathematica [A] time = 0.0775967, size = 126, normalized size = 0.98

$$
\frac{\sqrt{x\left(a+bx^2+cx^4\right)}\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c} - \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{2\sqrt{x}\sqrt{a+bx^2+cx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x*(a + b*x^2 + c*x^4)]*(((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c)
 - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])
])/(8*c^(3/2))))/(2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.016, size = 157, normalized size = 1.2

$$
\frac{1}{16}\sqrt{x\left(cx^4+bx^2+a\right)}\left(4\,x^2c^{3/2}\sqrt{cx^4+bx^2+a}+4\,\ln\left(1/2\,\frac{2\,cx^2+2\,\sqrt{cx^4+bx^2+a}\sqrt{c}+b}{\sqrt{c}}\right)ac-\ln\left(\frac{1}{2}\left(2\,cx^2+2\,\sqrt{cx^4}\right.\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/16*(x*(c*x^4+b*x^2+a))^(1/2)/c^(3/2)*(4*x^2*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)
+4*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a*c-ln(1/2*(
```

```
2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*b^2+2*b*(c*x^4+b*x^2+a)
^(1/2)*c^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^5 + bx^3 + ax}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x), x)
```

Fricas [A] time = 1.39673, size = 547, normalized size = 4.24

$$\left[-\frac{\left(b^2 - 4\,ac\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^5 + 8\,bcx^3 + 4\,\sqrt{cx^5 + bx^3 + ax}\left(2\,cx^2 + b\right)\sqrt{c}\sqrt{x} + \left(b^2 + 4\,ac\right)x}{x}\right) - 4\,\sqrt{cx^5 + bx^3 + ax}\left(2\,c^2x^2 + bc\right)\sqrt{x}}{32\,c^2x}, \frac{\left(b^2 - 4\,ac\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5
+ b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) - 4*sqrt
(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x))/(c^2*x), 1/16*((b^2 - 4*a*
c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*s
qrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^
2 + b*c)*sqrt(x))/(c^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\sqrt{x\left(a + bx^2 + cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.19448, size = 171, normalized size = 1.33

$$\frac{1}{8}\sqrt{cx^4 + bx^2 + a}\left(2\,x^2 + \frac{b}{c}\right) + \frac{\left(b^2 - 4\,ac\right)\log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{16\,c^{\frac{3}{2}}} - \frac{b^2\log\left(\left|-b + 2\,\sqrt{a}\sqrt{c}\right|\right) - 4\,ac\log}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(-2*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2) - 1/16*(b^2*lo
g(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2
*sqrt(a)*b*sqrt(c))/c^(3/2)
```

3.107 $\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$

Optimal. Leaf size=347

$$\frac{\sqrt[4]{a}\sqrt{x}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{a}b\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+c}{\left(\sqrt{a}+\sqrt{c}\right.}}}{3c^{3/4}\sqrt{ax}}$$

```
[Out] (b*x^(3/2)*(a + b*x^2 + c*x^4))/(3*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x
 + b*x^3 + c*x^5]) + (Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])/3 - (a^(1/4)*b*Sqr
t[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(3*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*
Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(6*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.224418, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1921, 1953, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\sqrt{x}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{a}b\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\,\text{t}\right.}{3c^{3/4}\sqrt{ax+bx^3+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

```
[Out] (b*x^(3/2)*(a + b*x^2 + c*x^4))/(3*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x
 + b*x^3 + c*x^5]) + (Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])/3 - (a^(1/4)*b*Sqr
t[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(3*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*
Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(6*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
```

```
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}+\frac{1}{3}\int \frac{\sqrt{x}\left(2a+bx^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx$$

$$= \frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}+\frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}+\frac{\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{ax+bx^3+cx^5}}-\frac{\left(\sqrt{a}b\sqrt{x}\sqrt{a\right.}{3\ldots}$$

$$= \frac{bx^{3/2}\left(a+bx^2+cx^4\right)}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5}-\frac{\sqrt[4]{a}b\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2\ldots}{(\sqrt{a}+\sqrt{\ldots}}}}{3c^{3/4}\sqrt{\ldots}}$$

Mathematica [C] time = 0.970893, size = 452, normalized size = 1.3

$$\frac{\sqrt{x}\left(-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)\ldots}{12c\sqrt{\frac{\ldots}{\sqrt{b^2-\ldots}}}\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*(
-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
```

```
2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2
- 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elliptic
F[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])]))/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.019, size = 508, normalized size = 1.5

$$\frac{1}{3\,cx^4+3\,bx^2+3\,a}\sqrt{x\left(cx^4+bx^2+a\right)}\left(\sqrt{-4\,ac+b^2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}x^5c+\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}x^5bc+\sqrt{-4\,ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x)
```

```
[Out] 1/3*(x*(c*x^4+b*x^2+a))^(1/2)/x^(1/2)*((-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+
b^2)^(1/2)))^(1/2)*x^5*c+(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^5*b*c+(-4*a*
c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*b+(1/a*(-b+(-4*a*c+b^2
)^(1/2)))^(1/2)*x^3*b^2+a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*(
(x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-
b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/
a/c)^(1/2))*(-4*a*c+b^2)^(1/2)+b*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a
)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2
)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*
a*c+b^2)/a/c)^(1/2))+(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)
*x*a+(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*a*b)/(c*x^4+b*x^2+a)/(1/a*(-b+(-
4*a*c+b^2)^(1/2)))^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5+bx^3+ax}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(a+bx^2+cx^4\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/sqrt(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5+bx^3+ax}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

3.108 $\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}-\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) - (Sqrt[a]*Sqrt[x]*Sqrt[a + b*x^2 + c
*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a
*x + b*x^3 + c*x^5]) + (b*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*Sqrt[a*x + b*x^3 + c*
x^5])
```

Rubi [A] time = 0.208964, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1921, 1953, 1251, 843, 621, 206, 724}

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}-\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

```
[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) - (Sqrt[a]*Sqrt[x]*Sqrt[a + b*x^2 + c
*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a
*x + b*x^3 + c*x^5]) + (b*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*Sqrt[a*x + b*x^3 + c*
x^5])
```

Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx &= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{1}{2}\int \frac{2a+bx^2}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx \\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{2a+bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx}{2\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\mathrm{Subst}\left(\int \frac{2a+bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{\left(a\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2\sqrt{ax+bx^3+cx^5}} + \frac{\left(b\sqrt{x}\sqrt{a+bx^2+\right.}{} \\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\left(a\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax+bx^3+cx^5}} + \frac{\left(b\sqrt{x}\sqrt{a+bx\right.}{} \\
&= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\tan}{4\sqrt{c}\sqrt{ax+b}}
\end{aligned}
$$

Mathematica [A] time = 0.0663108, size = 155, normalized size = 0.8

$$
\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}-2\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\right)}{4\sqrt{c}\sqrt{x\left(a+bx^2+cx^4\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4] - 2*Sqr
t[a]*Sqrt[c]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] + b
*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(4*Sqrt[c]*Sq
rt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.017, size = 136, normalized size = 0.7

$$-\frac{1}{4}\sqrt{x\left(cx^4+bx^2+a\right)}\left(2\sqrt{a}\ln\left(\frac{2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}}{x^2}\right)\sqrt{c}-b\ln\left(\frac{1}{2}\left(2cx^2+2\sqrt{cx^4+bx^2+a}\sqrt{c}+b\right)\frac{1}{\sqrt{c}}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x)
```

```
[Out] -1/4*(x*(c*x^4+b*x^2+a))^(1/2)*(2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*
x^2+a)^(1/2))/x^2)*c^(1/2)-b*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2
)+b)/c^(1/2))-2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2
)/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5+bx^3+ax}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2), x)
```

Fricas [A] time = 1.64065, size = 1575, normalized size = 8.12

$$\left[\frac{b\sqrt{cx}\log\left(-\frac{8c^2x^5+8bcx^3+4\sqrt{cx^5+bx^3+ax}\left(2cx^2+b\right)\sqrt{c}\sqrt{x}+\left(b^2+4ac\right)x}{x}\right)+2\sqrt{a}cx\log\left(-\frac{\left(b^2+4ac\right)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}\left(bx^2+2a\right)\sqrt{a}\sqrt{x}}{x^5}\right.}{8cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)
*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(
(b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^
2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*
x), -1/4*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*s
qrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - sqrt(a)*c*x*log(-((b^2 + 4*a
```

```
*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*s
qrt(a)*sqrt(x))/x^5) - 2*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/8*(4
*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*s
qrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x
^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a
*c)*x)/x) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/4*(2*sqrt(-a)*c
*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c
*x^5 + a*b*x^3 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x
)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^
5 + b*x^3 + a*x)*c*sqrt(x))/(c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(a+bx^2+cx^4\right)}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/x**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5+bx^3+ax}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2), x)
```

3.109 $\int x^{3/2} \left(ax+bx^3+cx^5\right)^{3/2} \, dx$

Optimal. Leaf size=244

$$\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{640c^2}-\frac{3b\sqrt{x}\left(b^2-4\right.}{}$$

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5])/(1280*c^3*
Sqrt[x]) - (x^(3/2)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 16*a*c)*x^2)*Sqrt[a*x
 + b*x^3 + c*x^5])/(640*c^2) + (Sqrt[x]*(3*b + 8*c*x^2)*(a*x + b*x^3 + c*x^
5)^(3/2))/(80*c) - (3*b*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*Arc
Tanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*Sqrt[
a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.357206, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1919, 1945, 1949, 12, 1914, 1107, 621, 206}

$$\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{640c^2}-\frac{3b\sqrt{x}\left(b^2-4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5])/(1280*c^3*
Sqrt[x]) - (x^(3/2)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 16*a*c)*x^2)*Sqrt[a*x
 + b*x^3 + c*x^5])/(640*c^2) + (Sqrt[x]*(3*b + 8*c*x^2)*(a*x + b*x^3 + c*x^
5)^(3/2))/(80*c) - (3*b*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*Arc
Tanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*Sqrt[
a*x + b*x^3 + c*x^5])
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
```

```
(n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}dx=\frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}+\frac{3\int\sqrt{x}\left(-2ab-\left(5b^2-16ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{80c}$$

$$=-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2-16ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{640c^2}+\frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3\right.}{80c}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2-16ac\right)x^2\right)}{640c^2}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2-16ac\right)x^2\right)}{640c^2}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2-16ac\right)x^2\right)}{640c^2}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2-16ac\right)x^2\right)}{640c^2}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2-16ac\right)x^2\right)}{640c^2}$$

$$=\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2-16ac\right)x^2\right)}{640c^2}$$

Mathematica [A] time = 0.212016, size = 192, normalized size = 0.79

$$\frac{\left(x\left(a+bx^2+cx^4\right)\right)^{3/2}\left(-\frac{3b\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\right)}{256c^{7/2}}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c^2}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{5c}\right)}{2x^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((x*(a + b*x^2 + c*x^4))^(3/2)*(-(b*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2)
)/(16*c^2) + (a + b*x^2 + c*x^4)^(5/2)/(5*c) - (3*b*(b^2 - 4*a*c)*(-2*Sqrt[
c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(256*c^(7/2))))/(2*x^(3/2)*(a +
b*x^2 + c*x^4)^(3/2))
```

Maple [A] time = 0.026, size = 369, normalized size = 1.5

$$-\frac{1}{2560}\sqrt{x\left(cx^4+bx^2+a\right)}\left(-256\,x^8c^{9/2}\sqrt{cx^4+bx^2+a}-352\,x^6bc^{7/2}\sqrt{cx^4+bx^2+a}-512\,x^4ac^{7/2}\sqrt{cx^4+bx^2+a}-16\,x^4b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] -1/2560*(x*(c*x^4+b*x^2+a))^(1/2)/c^(7/2)*(-256*x^8*c^(9/2)*(c*x^4+b*x^2+a)
^(1/2)-352*x^6*b*c^(7/2)*(c*x^4+b*x^2+a)^(1/2)-512*x^4*a*c^(7/2)*(c*x^4+b*x
```

```
^2+a)^(1/2)-16*x^4*b^2*c^(5/2)*(c*x^4+b*x^2+a)^(1/2)-112*x^2*a*b*c^(5/2)*(c
*x^4+b*x^2+a)^(1/2)+20*x^2*b^3*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)+240*ln(1/2*(2*
c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a^2*b*c^2-120*ln(1/2*(2*c
*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a*b^3*c+15*ln(1/2*(2*c*x^2
+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*b^5-256*a^2*c^(5/2)*(c*x^4+b*x
^2+a)^(1/2)+200*a*b^2*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)-30*b^4*c^(1/2)*(c*x^4+b
*x^2+a)^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}} x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2), x)
```

Fricas [A] time = 1.44359, size = 921, normalized size = 3.77

$$\left[\frac{15\left(b^5 - 8\,ab^3c + 16\,a^2bc^2\right)\sqrt{cx}\log\left(-\frac{8\,c^2x^5+8\,bcx^3-4\,\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{c}\sqrt{x}+\left(b^2+4\,ac\right)x}{x}\right) + 4\left(128\,c^5x^8 + 176\,bc^4x^6 + 15\,b^4\right.}{5120\,c^4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*
b*c*x^3 - 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2
+ 4*a*c)*x)/x) + 4*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2
+ 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2
)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*x), 1/2560*(15*(b^5 - 8*a*b^3*c +
 16*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b
)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*(128*c^5*x^8 + 176*b*c^
4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4
 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*
x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}} x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2), x)
```

3.110 $\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx$

Optimal. Leaf size=487

$$\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right.\right.}{630c^{11/4}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x^(3/2)*(a + b*x^2 + c*x^4))/(315*c^(5/2
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (Sqrt[x]*(b*(4*b^2 -
 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(315*c^2) + (
(3*b + 7*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(63*c*Sqrt[x]) - (a^(1/4)*(8*b
^4 - 57*a*b^2*c + 84*a^2*c^2)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a*x + b*x^3 + c*x^5])
 + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[a]*b*Sqrt[c]*(b^2 - 6
*a*c))*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(630*c^(11/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.457044, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1919, 1945, 1953, 1197, 1103, 1195}

$$\frac{x^{3/2}\left(84a^2c^2-57ab^2c+8b^4\right)\left(a+bx^2+cx^4\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{}}{630c^{11/4}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```


```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x^(3/2)*(a + b*x^2 + c*x^4))/(315*c^(5/2
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (Sqrt[x]*(b*(4*b^2 -
 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(315*c^2) + (
(3*b + 7*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(63*c*Sqrt[x]) - (a^(1/4)*(8*b
^4 - 57*a*b^2*c + 84*a^2*c^2)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a*x + b*x^3 + c*x^5])
 + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[a]*b*Sqrt[c]*(b^2 - 6
*a*c))*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(630*c^(11/4)*Sqrt[a*x + b*x^3 + c*x^5])
```


Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
 + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
```

```
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}dx = \frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}} + \frac{\int \frac{\left(-ab-2\left(2b^2-7ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}dx}{21c}$$

$$= -\frac{\sqrt{x}\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2} + \frac{\left(3b+7cx^2\right)\left(ax+bx^3+c\right.}{63c\sqrt{x}}$$

$$= -\frac{\sqrt{x}\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2} + \frac{\left(3b+7cx^2\right)\left(ax+bx^3+c\right.}{63c\sqrt{x}}$$

$$= -\frac{\sqrt{x}\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2} + \frac{\left(3b+7cx^2\right)\left(ax+bx^3+c\right.}{63c\sqrt{x}}$$

$$= \frac{\left(8b^4-57ab^2c+84a^2c^2\right)x^{3/2}\left(a+bx^2+cx^4\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt{x}\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)}{315c^2}$$

Mathematica [C] time = 2.32099, size = 609, normalized size = 1.25

$$\frac{\sqrt{x}\left(-i\left(84a^2c^2\sqrt{b^2-4ac}-132a^2bc^2+8b^4\sqrt{b^2-4ac}+65ab^3c-57ab^2c\sqrt{b^2-4ac}-8b^5\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}}{b-\sqrt{b^2}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3*c*x^4 + 5
3*b^2*c^2*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*x^2) + a*(-
4*b^3 + 27*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) + I*(8*b^4 - 57*a*b^2*
c + 84*a^2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^5 +
65*a*b^3*c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*Sqrt[b^2
- 4*a*c] + 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(1260*c^3*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] time = 0.024, size = 1878, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x)
```

```
[Out] -1/315*(x*(c*x^4+b*x^2+a))^(1/2)*(-151*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*
(-4*a*c+b^2)^(1/2)*x^5*a*b*c^2+84*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)
```

```
^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)
*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a
*c+b^2)/a/c)^(1/2))*a^3*c^2+6*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/
2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/
a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b
^2)/a/c)^(1/2))*a*b^4-84*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((
x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*(1/a*(-b
+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a
/c)^(1/2))*a^3*c^2-8*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*
(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*(1/a*(-b+(-4
*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^
(1/2))*a*b^4-24*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*a^2*b^2*c-151*(1/a*(-
b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^5*a*b^2*c^2-77*(1/a*(-b+(-4*a*c+b^2)^(1/2)))
^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*a^2*c^2-77*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2
)*x^3*a^2*b*c^2-27*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*a*b^3*c-53*(1/a*
(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^7*b^2*c^2-112*(1/a*(-b+
(-4*a*c+b^2)^(1/2)))^(1/2)*x^7*a*b*c^3+(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*
(-4*a*c+b^2)^(1/2)*x^5*b^3*c-85*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c
+b^2)^(1/2)*x^9*b*c^3-112*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^
(1/2)*x^7*a*c^3+4*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*b^5+4*(1/a*(-b+(-
4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x*a*b^3+12*(-2*(x^2*(-4*a*c+b^2
)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(
-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)*a^2*b*c-45*(-2*
(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+
2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1
/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*b^2*c+57*(-2*(
x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2
*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/
2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*b^2*c-2*(-2*(x^
2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a
)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*
2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)*a*
b^3-27*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*a*b^2*c-2
4*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x*a^2*b*c-85*(1/a*
(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^9*b^2*c^3-53*(1/a*(-b+(-4*a*c+b^2)^(1/2)))
^(1/2)*x^7*b^3*c^2+(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^5*b^4*c+4*(1/a*(-b
+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b^4+4*(1/a*(-b+(-4*a*c+b
^2)^(1/2)))^(1/2)*x*a*b^4-35*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^
2)^(1/2)*x^11*c^4-35*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^11*b*c^4)/x^(1/2
)/(c*x^4+b*x^2+a)/c^2/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b+(-4*a*c+b^2)^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}} \sqrt{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^5+bx^3+ax\right)^{\frac{3}{2}}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

3.111 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}} dx$

Optimal. Leaf size=177

$$-\frac{3(b^2-4ac)(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}}+\frac{3\sqrt{x}(b^2-4ac)^2\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}}+\frac{(b+2cx^2)(ax+}{16cx^{3/}}$$

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(128*c^2*Sqrt[x]
) + ((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(16*c*x^(3/2)) + (3*(b^2 -
4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.138344, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1918, 1914, 1107, 621, 206}

$$-\frac{3(b^2-4ac)(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}}+\frac{3\sqrt{x}(b^2-4ac)^2\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}}+\frac{(b+2cx^2)(ax+}{16cx^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(128*c^2*Sqrt[x]
) + ((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(16*c*x^(3/2)) + (3*(b^2 -
4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{\sqrt{x}}\,dx = \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} - \frac{\left(3\left(b^2-4ac\right)\right)\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx}{16c}$$

$$= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{\left(3\left(b^2-4a\right.\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{\left(3\left(b^2-4a\right.\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{\left(3\left(b^2-4a\right.\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{\left(3\left(b^2-4a\right.\right.}{}$$

$$= -\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}} + \frac{3\left(b^2-4ac\right.}{}$$

Mathematica [A] time = 0.113543, size = 152, normalized size = 0.86

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(4c\left(5a+2cx^4\right)-3b^2+8bcx^2\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+}}\right.\right.}{256c^{5/2}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 +
c*x^4]*(-3*b^2 + 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^2 - 4*a*c)^2*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/(256*c^(5/2)*Sqrt[x
*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.02, size = 295, normalized size = 1.7

$$\frac{1}{256}\sqrt{x\left(cx^4+bx^2+a\right)}\left(32\,x^6c^{7/2}\sqrt{cx^4+bx^2+a}+48\,x^4bc^{5/2}\sqrt{cx^4+bx^2+a}+80\,x^2ac^{5/2}\sqrt{cx^4+bx^2+a}+4\,x^2b^2c^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x)
```

```
[Out] 1/256*(x*(c*x^4+b*x^2+a))^(1/2)/c^(5/2)*(32*x^6*c^(7/2)*(c*x^4+b*x^2+a)^(1/
2)+48*x^4*b*c^(5/2)*(c*x^4+b*x^2+a)^(1/2)+80*x^2*a*c^(5/2)*(c*x^4+b*x^2+a)^
(1/2)+4*x^2*b^2*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)+48*ln(1/2*(2*c*x^2+2*(c*x^4+b
*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a^2*c^2-24*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2
+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a*b^2*c+3*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(
1/2)*c^(1/2)+b)/c^(1/2))*b^4+40*a*b*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)-6*b^3*c^(
1/2)*(c*x^4+b*x^2+a)^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/sqrt(x), x)
```

Fricas [A] time = 1.51134, size = 768, normalized size = 4.34

$$\left[\frac{3\left(b^4 - 8\,ab^2c + 16\,a^2c^2\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{c}\sqrt{x}+\left(b^2+4\,ac\right)x}{x}\right) + 4\left(16\,c^4x^6 + 24\,bc^3x^4 - 3\,b^3c + 20\,ab\right.}{512\,c^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*
x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*
a*c)*x)/x) + 4*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c
^2 + 20*a*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x), -1/256*(3*(
b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*
x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - 2*(16*c^4*
x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*sqr
t(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.5184, size = 624, normalized size = 3.53

$$\frac{1}{16}\left(2\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{\left(b^2-4ac\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}-\frac{b^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*log(abs(-2*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2) - (b^2*log(abs(
-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2*sqrt(
a)*b*sqrt(c))/c^(3/2))*a + 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)
*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - s
qrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2) + (3*b^3*log(abs(-b + 2*sqrt(
a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sq
rt(c) - 16*a^(3/2)*c^(3/2))/c^(5/2))*b + 1/384*(sqrt(c*x^4 + b*x^2 + a)*(2*
(4*(6*x^2 + b/c)*x^2 - (5*b^2*c^7 - 12*a*c^8)/c^9)*x^2 + (15*b^3*c^6 - 52*a
*b*c^7)/c^9) - (15*sqrt(a)*b^3 - 52*a^(3/2)*b*c)/c^3)*c + 1/256*(5*b^4*c^6
- 24*a*b^2*c^7 + 16*a^2*c^8)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
 a))*sqrt(c) - b))/c^(17/2)
```

3.112 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=425

$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}-\frac{2bx^{3/}}{35c^{3/2}(}$$

```
[Out] (-2*b*(b^2 - 8*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(35*c^(3/2)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b^2 + 10*a*c + 3*b*c*x^2)*
Sqrt[a*x + b*x^3 + c*x^5])/(35*c) + (a*x + b*x^3 + c*x^5)^(3/2)/(7*Sqrt[x])
 + (2*a^(1/4)*b*(b^2 - 8*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5]) -
 (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*Sqrt[x]*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(70*c^(
7/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.44855, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1921, 1945, 1953, 1197, 1103, 1195}

$$-\frac{2bx^{3/2}\left(b^2-8ac\right)\left(a+bx^2+cx^4\right)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\right.}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```


```
[Out] (-2*b*(b^2 - 8*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(35*c^(3/2)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b^2 + 10*a*c + 3*b*c*x^2)*
Sqrt[a*x + b*x^3 + c*x^5])/(35*c) + (a*x + b*x^3 + c*x^5)^(3/2)/(7*Sqrt[x])
 + (2*a^(1/4)*b*(b^2 - 8*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5]) -
 (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*Sqrt[x]*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(70*c^(
7/4)*Sqrt[a*x + b*x^3 + c*x^5])
```


Rule 1921

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(b*B*(n - q)*p +
 A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{x^{3/2}}\,dx = \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}} + \frac{3}{7}\int \frac{\left(2a+bx^2\right)\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$

$$= \frac{\sqrt{x}\left(b^2+10ac+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}} + \frac{\int \frac{\sqrt{x}\left(-a\left(b^2-20ac\right)-2b\left(b^2-8ac\right)x^{2}\cdots\right)}{\sqrt{ax+bx^3+cx^5}}}{35c}\cdots$$

$$= \frac{\sqrt{x}\left(b^2+10ac+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}} + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{-a(\cdots}{\cdots}}{35c\sqrt{ax+b\cdots}}\cdots$$

$$= \frac{\sqrt{x}\left(b^2+10ac+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}} + \frac{\left(2\sqrt{a}b\left(b^2-8ac\right)\sqrt{x}\sqrt{a+\cdots}\right.}{35c^{3/2}\sqrt{ax\cdots}}\cdots$$

$$= -\frac{2b\left(b^2-8ac\right)x^{3/2}\left(a+bx^2+cx^4\right)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt{x}\left(b^2+10ac+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{\left(ax+b\cdots\right.}{\cdots}\cdots$$

Mathematica [C] time = 1.7485, size = 540, normalized size = 1.27

$$\frac{\sqrt{x}\left(i\left(-20a^2c^2+b^3\sqrt{b^2-4ac}+9ab^2c-8abc\sqrt{b^2-4ac}-b^4\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

```
[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*c
*x^2 + 20*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6)) -
I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 +
9*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
- 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSi
nh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])]))/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a +
b*x^2 + c*x^4)])
```

Maple [B] time = 0.023, size = 1394, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x)
```

```
[Out] -1/70*(x*(c*x^4+b*x^2+a))^(1/2)*(-10*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^
9*b*c^3-10*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^9*c^3-2
6*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^7*b^2*c^2-26*(-4*a*c+b^2)^(1/2)*(1/
a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^7*b*c^2-40*(1/a*(-b+(-4*a*c+b^2)^(1/2)))
```

```
^(1/2)*x^5*a*b*c^2-40*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2
)*x^5*a*c^2-18*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^5*b^3*c-18*(-4*a*c+b^2
)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^5*b^2*c-46*(1/a*(-b+(-4*a*c+b
^2)^(1/2)))^(1/2)*x^3*a*b^2*c-46*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(
1/2)))^(1/2)*x^3*a*b*c-2*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*b^4-2*(-4*
a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*b^3+12*EllipticF(1/2
*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)
^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/
2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a^2*b*c-20*(-4*a*c+b^2)^(1/
2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*
((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b
*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a^2*c-3*Ell
ipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-
4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2
*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*b^3+(-4*a*c+b^2
)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(
1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1
/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*b^2-
32*EllipticE(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*
((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b
*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a^2*b*c+4*E
llipticE(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*
(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2
-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*b^3-30*(1/a*(
-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*a^2*b*c-30*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*
a*c+b^2)^(1/2)))^(1/2)*x*a^2*c-2*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*a*b^
3-2*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*a*b^2)/x^(1/2)
/(c*x^4+b*x^2+a)/c/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b+(-4*a*c+b^2)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5 + bx^3 + ax}\left(cx^4 + bx^2 + a\right)}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*(c*x^4 + b*x^2 + a)/sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(3/2),x)
```

```
[Out] Integral((x*(a + b*x**2 + c*x**4))**(3/2)/x**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

3.113 $\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.0618114, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1914, 1107, 621, 206}

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}$$
$$= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2\sqrt{ax+bx^3+cx^5}}$$
$$= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax+bx^3+cx^5}}$$
$$= \frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Mathematica [A] time = 0.0182489, size = 82, normalized size = 1.

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.012, size = 72, normalized size = 0.9

$$\frac{1}{2}\sqrt{x\left(cx^4+bx^2+a\right)}\ln\left(\frac{1}{2}\left(2\,cx^2+2\,\sqrt{cx^4+bx^2+a}\sqrt{c}+b\right)\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{cx^4+bx^2+a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*ln(1/2*(2*c*x^2
+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [A] time = 1.46885, size = 325, normalized size = 3.96

$$\left[\frac{\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{c}\sqrt{x}+\left(b^2+4\,ac\right)x}{x}\right)}{4\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{-c}\sqrt{x}}{2\left(c^2x^5+bcx^3+acx\right)}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b
)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*s
qrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3
+ a*c*x))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.31361, size = 54, normalized size = 0.66

$$-\frac{\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{1024\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/1024*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^
(3/2)
```

3.114 $\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.0463985, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1914, 1103}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx &= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}\\ &= \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}\end{aligned}$$

Mathematica [C] time = 0.126265, size = 193, normalized size = 1.6

$$-\frac{i\sqrt{x}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] ((-I)*Sqrt[x]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x
^4)])
```

Maple [A] time = 0.016, size = 177, normalized size = 1.5

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\sqrt{x\left(cx^4+bx^2+a\right)}\sqrt{-2\,\frac{x^2\sqrt{-4\,ac+b^2}-bx^2-2\,a}{a}}\sqrt{\frac{1}{a}\left(x^2\sqrt{-4\,ac+b^2}+bx^2+2\,a\right)}\text{EllipticF}\left(\frac{x\sqrt{}}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)/(1/a*(-b+(-4*a*c+b^2)
^(1/2)))^(1/2)*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*
c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b
^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.115 $\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*
Sqrt[a])
```

Rubi [A] time = 0.0293618, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*
Sqrt[a])
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx &= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}\left(2a+bx^2\right)}{\sqrt{ax+bx^3+cx^5}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0193418, size = 83, normalized size = 1.63

$$-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] -(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
 b*x^2 + c*x^4])])/(2*Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.014, size = 72, normalized size = 1.4

$$-\frac{1}{2}\sqrt{x\left(cx^4+bx^2+a\right)}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{cx^4+bx^2+a}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] -1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*
a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^5+bx^3+ax}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)), x)
```

Fricas [A] time = 1.39996, size = 327, normalized size = 6.41

$$\left[\frac{\log\left(-\frac{\left(b^2+4\,ac\right)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}\left(bx^2+2\,a\right)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\,\sqrt{a}},\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\left(bx^2+2\,a\right)\sqrt{-a}\sqrt{x}}{2\left(acx^5+abx^3+a^2x\right)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 +
 a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*
sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3
 + a^2*x))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^5 + bx^3 + ax}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)), x)
```

3.116 $\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=330

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{a^{3/4}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x +
 b*x^3 + c*x^5]) - Sqrt[a*x + b*x^3 + c*x^5]/(a*x^(3/2)) - (c^(1/4)*Sqrt[x]
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a
^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*Sqrt[a*x + b*x
^3 + c*x^5])
```

Rubi [A] time = 0.184284, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1929, 12, 1914, 1139, 1103, 1195}

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\right.\right.}{a^{3/4}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```


```
[Out] (Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x +
 b*x^3 + c*x^5]) - Sqrt[a*x + b*x^3 + c*x^5]/(a*x^(3/2)) - (c^(1/4)*Sqrt[x]
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a
^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*Sqrt[a*x + b*x
^3 + c*x^5])
```


Rule 1929

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
)/(a*(m + p*q + 1)), x] - Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*(b*(m
 + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ
[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx &= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} + \frac{\int \frac{cx^{5/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{a} \\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} + \frac{c\int \frac{x^{5/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{a} \\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} + \frac{\left(c\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\sqrt{ax+bx^3+cx^5}} \\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} + \frac{\left(\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\sqrt{ax+bx^3+cx^5}} - \frac{\left(\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\cdots}{\sqrt{a}\sqrt{ax+bx^3}\cdots} \\
&= \frac{\sqrt{c}x^{3/2}\left(a+bx^2+cx^4\right)}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} - \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\cdots}{a^{3/4}\sqrt{ax+b\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.48716, size = 303, normalized size = 0.92

$$
\frac{-4\left(a+bx^2+cx^4\right)+\frac{i\sqrt{2}x\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{\cdots}{\sqrt{b^2-\cdots}}}\right.\right.\right.\cdots}{\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}}{4a\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (-4*(a + b*x^2 + c*x^4) + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[x]*S
qrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.023, size = 508, normalized size = 1.5

$$\frac{1}{\left(cx^4+bx^2+a\right)a}\left(-\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}\sqrt{-4\,ac+b^2}x^4c-\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}x^4bc-c\sqrt{-2\,\frac{x^2\sqrt{-4\,ac+b^2}-bx}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] (-(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^4*c-(1/a*(-b+(-4
*a*c+b^2)^(1/2)))^(1/2)*x^4*b*c-c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)
^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*x*EllipticF(1/2*x*2^(
1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)
-2*a*c+b^2)/a/c)^(1/2))+c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*(
(x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*x*EllipticE(1/2*x*2^(1/2)*(1/
a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b
^2)/a/c)^(1/2))-(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^2*
b-(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^2*b^2-(1/a*(-b+(-4*a*c+b^2)^(1/2)))
^(1/2)*(-4*a*c+b^2)^(1/2)*a-(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*a*b)/x^(3/2
)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)/a/(1/a*(-b+(-4*a*c+b^2)^(1/2)))
^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^5+bx^3+axx^{\frac{3}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}\sqrt{x}}{cx^7+bx^5+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c*x^7 + b*x^5 + a*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^5+bx^3+axx^{\frac{3}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

3.117 $\int \frac{x^{3/2}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\right.}{a^{3/4}\left(b^2-4ac\right)\sqrt{ax+bx^3}}$$

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5
]) - (b*Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (b*c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a
*c)*Sqrt[a*x + b*x^3 + c*x^5]) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sq
rt[c])*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.248059, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1924, 1953, 1197, 1103, 1195}

$$\frac{b\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5
]) - (b*Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (b*c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a
*c)*Sqrt[a*x + b*x^3 + c*x^5]) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sq
rt[c])*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
 && LtQ[m + p*q + 1, n - q]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{\sqrt{x}\left(2ac+bcx^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{2ac+bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} + \frac{\left(b\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{\left(\left(b+2\sqrt{a}\sqrt{c}\right.\right.}{\sqrt{\ }}$$

$$= \frac{x^{3/2}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{b\sqrt{c}x^{3/2}\left(a+bx^2+cx^4\right)}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}\right.}{}$$

Mathematica [C] time = 1.08292, size = 463, normalized size = 1.18

$$-\frac{\sqrt{x}\left(-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right.\right.}{4a\left(b^2-4ac\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] -(Sqrt[x]*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2) + I
*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elli
pticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(4*a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.027, size = 533, normalized size = 1.4

$$\frac{1}{\left(cx^4+bx^2+a\right)a\left(4ac-b^2\right)}\sqrt{x\left(cx^4+bx^2+a\right)}\left(-\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)}x^3b^2c-\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)}\sqrt{-4ac+b^2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] (x*(c*x^4+b*x^2+a))^(1/2)*(-(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*b^2*c-(
1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b*c+c*(-2*(x^2*(-
4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)
^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1
/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*(-4*a*c+b^2)^(1/2)+b*c*
(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*
x^2+2*a)/a)^(1/2)*a*EllipticE(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(
1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))+2*(1/a*(-b+(
-4*a*c+b^2)^(1/2)))^(1/2)*x*a*b*c+2*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4
*a*c+b^2)^(1/2)*x*a*c-(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*b^3-(1/a*(-b+(-
4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x*b^2)/x^(1/2)/(c*x^4+b*x^2+a)/
a/(4*a*c-b^2)/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}\sqrt{x}}{c^2x^9+2\,bcx^7+\left(b^2+2\,ac\right)x^5+2\,abx^3+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c^2*x^9 + 2*b*c*x^7 + (b^2 + 2*
a*c)*x^5 + 2*a*b*x^3 + a^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**(3/2)/(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

3.118 $\int \frac{\sqrt{x}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5
]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]
/(2*a^(3/2))
```

Rubi [A] time = 0.072923, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1922, 1913, 206}

$$\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5
]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]
/(2*a^(3/2))
```

Rule 1922

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q +
b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n
 - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, -((n - q)*(2*p + 3)
)]
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx}{a}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\right)}{a}$$

$$= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

Mathematica [A] time = 0.0829563, size = 126, normalized size = 1.22

$$\frac{\sqrt{x}\left(\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-2\sqrt{a}\left(-2ac+b^2+bcx^2\right)\right)}{2a^{3/2}\left(4ac-b^2\right)\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(-2*Sqrt[a]*(b^2 - 2*a*c + b*c*x^2) + (b^2 - 4*a*c)*Sqrt[a + b*x^2
 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]))/(2*a
^(3/2)*(-b^2 + 4*a*c)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] time = 0.019, size = 179, normalized size = 1.7

$$-\frac{1}{\left(2\,cx^4+2\,bx^2+2\,a\right)\left(4\,ac-b^2\right)}\sqrt{x\left(cx^4+bx^2+a\right)}\left(2\,x^2bc\sqrt{a}+4\,\ln\left(\frac{2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}}{x^2}\right)ac\sqrt{cx^4+bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] -1/2*(x*(c*x^4+b*x^2+a))^(1/2)/a^(3/2)*(2*x^2*b*c*a^(1/2)+4*ln((2*a+b*x^2+2
*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*a*c*(c*x^4+b*x^2+a)^(1/2)-ln((2*a+b*x^
2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*b^2*(c*x^4+b*x^2+a)^(1/2)-4*a^(3/2)
*c+2*b^2*a^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

Fricas [B] time = 1.7769, size = 918, normalized size = 8.91

$$\left[\frac{\left(\left(b^2c - 4\,ac^2\right)x^5 + \left(b^3 - 4\,abc\right)x^3 + \left(ab^2 - 4\,a^2c\right)x\right)\sqrt{a}\log\left(-\frac{\left(b^2+4\,ac\right)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}\left(bx^2+2\,a\right)\sqrt{a}\sqrt{x}}{x^5}\right) + 4\,\sqrt{cx^5 + b}}{4\left(\left(a^2b^2c - 4\,a^3c^2\right)x^5 + \left(a^2b^3 - 4\,a^3bc\right)x^3 + \left(a^3b^2 - 4\,a^4c\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*s
qrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3
 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*(
a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b
^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x), 1/2*(((b^2*c - 4*a*c^2)*x^5 +
 (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^5
+ b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x))
+ 2*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*
b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(x\left(a + bx^2 + cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

3.119 $\int \frac{1}{\sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

Optimal. Leaf size=468

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}+\frac{2\sqrt{c}x^{3/2}\left(b^2-3\right.}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\right.}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])
 + (2*Sqrt[c]*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(a^2*(b^2 - 4*a*c)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (2*(b^2 - 3*a*c)*Sqrt
[a*x + b*x^3 + c*x^5])/(a^2*(b^2 - 4*a*c)*x^(3/2)) - (2*c^(1/4)*(b^2 - 3*a*
c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*(2*b^2
 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^
3 + c*x^5])
```

Rubi [A] time = 0.408659, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1924, 1951, 1953, 1197, 1103, 1195}

$$\frac{2\sqrt{c}x^{3/2}\left(b^2-3ac\right)\left(a+bx^2+cx^4\right)}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2x^{3/2}\left(b^2-4ac\right)}+\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x\right.}{2a^{7/4}\left(b^2-4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```


```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])
 + (2*Sqrt[c]*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(a^2*(b^2 - 4*a*c)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (2*(b^2 - 3*a*c)*Sqrt
[a*x + b*x^3 + c*x^5])/(a^2*(b^2 - 4*a*c)*x^(3/2)) - (2*c^(1/4)*(b^2 - 3*a*
c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*(2*b^2
 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^
3 + c*x^5])
```


Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
```

```
&& LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{-2b^2+6ac-bcx^2}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2\left(b^2-4ac\right)x^{3/2}} + \frac{\int \frac{\sqrt{x}\left(abc+2c\left(b^2-3ac\right)x\right.}{\sqrt{ax+bx^3+cx^5}}}{a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2\left(b^2-4ac\right)x^{3/2}} + \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx\right.}{a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}} - \frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2\left(b^2-4ac\right)x^{3/2}} - \frac{\left(2\sqrt{c}\left(b^2-3ac\right)\sqrt{\right.}}{a^{3/2}\left(b^2\right.}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}} + \frac{2\sqrt{c}\left(b^2-3ac\right)x^{3/2}\left(a+bx^2+cx^4\right)}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} - \frac{2\left(b\right.}{}$$

Mathematica [C] time = 1.35104, size = 519, normalized size = 1.11

$$-\frac{ix\left(b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}+4abc-b^3\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac+}}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] -(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*x^2) + a*(
b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*x*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqr
t[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] time = 0.029, size = 1136, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x)
```

```
[Out] -1/2*(x*(c*x^4+b*x^2+a))^(1/2)/x^(3/2)*(12*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1
/2)*(-4*a*c+b^2)^(1/2)*x^4*a*c^2-4*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*
a*c+b^2)^(1/2)*x^4*b^2*c+12*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^4*a*b*c^2
```

```
-4*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^4*b^3*c+a*b*c*(-2*(x^2*(-4*a*c+b^2
)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(
-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x*(-4*a*c+b^2)^(1/2)+12*(-2*(x^2*(
-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a
)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(
1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x*a^2*c^2-3*a*b^2*c*(-2*
(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+
2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1
/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x-12*(-2*(x^2*(-4*
a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(
1/2)*EllipticE(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2
)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x*a^2*c^2+4*(-2*(x^2*(-4*a*
c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/
2)*EllipticE(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*
((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x*a*b^2*c+14*(1/a*(-b+(-4*a*c
+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^2*a*b*c-4*(1/a*(-b+(-4*a*c+b^2)^(1
/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^2*b^3+14*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/
2)*x^2*a*b^2*c-4*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^2*b^4+8*(1/a*(-b+(-4
*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*a^2*c-2*(1/a*(-b+(-4*a*c+b^2)^(1
/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*a*b^2+8*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*
a^2*b*c-2*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*a*b^3)/(c*x^4+b*x^2+a)/a^2/(4
*a*c-b^2)/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}\sqrt{x}}{c^2x^{11}+2\,bcx^9+\left(b^2+2\,ac\right)x^7+2\,abx^5+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c^2*x^11 + 2*b*c*x^9 + (b^2 + 2
*a*c)*x^7 + 2*a*b*x^5 + a^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}} \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

3.120 $\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2x^{5/2}\left(b^2-4ac\right)}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
 - ((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(b^2 - 4*a*c)*x^(5/2)
) + (3*b*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^
5])])/(4*a^(5/2))
```

Rubi [A] time = 0.174953, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1924, 1951, 12, 1913, 206}

$$-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2x^{5/2}\left(b^2-4ac\right)}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```



```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
 - ((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(b^2 - 4*a*c)*x^(5/2)
) + (3*b*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^
5])])/(4*a^(5/2))
```


Rule 1924

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
 && LtQ[m + p*q + 1, n - q]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{-3b^2+8ac-2bcx^2}{x^{5/2}\sqrt{ax+bx^3+cx^5}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} + \frac{\int -\frac{3b\left(b^2-4ac\right)}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} - \frac{(3b)\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+\dots}}}{2a^2}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} + \frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{4a-\dots}\right.}{\dots}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{x}(\dots}{2\sqrt{a}\sqrt{a\dots}}\right.}{4a^{5/2}}$$

Mathematica [A] time = 0.0820778, size = 160, normalized size = 1.04

$$\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx^2-8c^2x^4\right)+3b^2x^2\left(b+cx^2\right)\right)-3bx^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}x^{3/2}\left(4ac-b^2\right)\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (2*Sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2*
x^4)) - 3*b*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)
/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*(-b^2 + 4*a*c)*x^(3/2)*Sq
rt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.019, size = 220, normalized size = 1.4

$$\frac{1}{\left(4\,c x^{4}+4\,b x^{2}+4\,a\right)\left(4\,ac-b^{2}\right)}\sqrt{x\left(c x^{4}+b x^{2}+a\right)}\left(-16\,x^{4}a^{3/2}c^{2}+6\,x^{4}b^{2}c\sqrt{a}+12\,\ln\left(\frac{2\,a+b x^{2}+2\,\sqrt{a}\sqrt{c x^{4}+b x^{2}+a}}{x^{2}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] 1/4*(x*(c*x^4+b*x^2+a))^(1/2)/a^(5/2)*(-16*x^4*a^(3/2)*c^2+6*x^4*b^2*c*a^(1
/2)+12*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*x^2*a*b*c*(c*x^4
+b*x^2+a)^(1/2)-3*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*x^2*b
^3*(c*x^4+b*x^2+a)^(1/2)-20*a^(3/2)*x^2*b*c+6*x^2*b^3*a^(1/2)-8*a^(5/2)*c+2
*a^(3/2)*b^2)/x^(5/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c x^{5}+b x^{3}+a x\right)^{\frac{3}{2}} x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)
```

Fricas [A] time = 1.85444, size = 1084, normalized size = 7.04

$$\left[\frac{3\left(\left(b^{3}c-4\,abc^{2}\right)x^{7}+\left(b^{4}-4\,ab^{2}c\right)x^{5}+\left(ab^{3}-4\,a^{2}bc\right)x^{3}\right)\sqrt{a}\log\left(-\frac{\left(b^{2}+4\,ac\right)x^{5}+8\,abx^{3}+8\,a^{2}x+4\,\sqrt{cx^{5}+bx^{3}+ax}\left(bx^{2}+2\,a\right)\sqrt{a}\sqrt{x}}{x^{5}}\right)-4\,\sqrt{c}}{8\left(\left(a^{3}b^{2}c-4\,a^{4}c^{2}\right)x^{7}+\left(a^{3}b^{3}-4\,a^{4}bc\right)x^{5}+\left(a^{4}b^{2}-4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b
*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x + 4*sqrt(c*x
^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) - 4*sqrt(c*x^5 + b*x^
3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a
^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*
x^5 + (a^4*b^2 - 4*a^5*c)*x^3), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4
*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b
*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + 2
*sqrt(c*x^5 + b*x^3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
 + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3
*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)
```

3.121 $\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^{n}+cx^{1+n}\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{\left(b^2-4ac\right)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^(-1 + n) + b*x^n +
c*x^(1 + n)])
```

Rubi [A] time = 0.0499673, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1915}

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{\left(b^2-4ac\right)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^(-1 + n) + b*x^n +
c*x^(1 + n)])
```

Rule 1915

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[(-2*x^((n - 1)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*
x^(n - 1) + b*x^n + c*x^(n + 1)]), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (
3*(n - 1))/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx=-\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{\left(b^2-4ac\right)\sqrt{ax^{-1+n}+bx^n+cx^{1+n}}}$$

Mathematica [A] time = 0.0883845, size = 46, normalized size = 0.9

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{\left(b^2-4ac\right)\sqrt{x^{n-1}(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^(-1 + n)*(a + x*(b +
c*x))])
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int x^{-\frac{3}{2}+\frac{3n}{2}} \left(a x^{-1+n} + b x^{n} + c x^{1+n}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x)
```

```
[Out] int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}n-\frac{3}{2}}}{\left(c x^{n+1} + a x^{n-1} + b x^{n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)
```

Fricas [A] time = 1.65691, size = 186, normalized size = 3.65

$$-\frac{2 \left(2 c x^{2} + b x\right) \sqrt{\frac{\left(c x^{2}+b x+a\right) x^{n+1}}{x^{2}}}}{\left(a b^{2} - 4 a^{2} c + \left(b^{2} c - 4 a c^{2}\right) x^{2} + \left(b^{3} - 4 a b c\right) x\right) x^{\frac{1}{2} n+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2*(2*c*x^2 + b*x)*sqrt((c*x^2 + b*x + a)*x^(n + 1)/x^2)/((a*b^2 - 4*a^2*c
+ (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*x^(1/2*n + 1/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3/2+3/2*n)/(a*x**(-1+n)+b*x**n+c*x**(1+n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}n-\frac{3}{2}}}{\left(cx^{n+1}+ax^{n-1}+bx^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)
```

3.122 $\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=287

$$\frac{2dx^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{7}{4}\right.}{7\sqrt{ax+bx^3}}$$

```
[Out] (2*d*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^
5]) + (2*e*x^4*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*Sqrt[a*x + b*x^3
 + c*x^5])
```

Rubi [A] time = 0.399178, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1954, 1335, 1141, 510}

$$\frac{2dx^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{7}{4}\right.}{7\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*d*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^
5]) + (2*e*x^4*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*Sqrt[a*x + b*x^3
 + c*x^5])
```

Rule 1954

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^(p_
)*((A_) + (B_.)*(x_)^(q_)), x_Symbol] :> Dist[(a*x^j + b*x^k + c*x^n)^p/(x^
(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p), Int[x^(m + j*p)*(A + B*x^(k -
 j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B,
 j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] &&  !IntegerQ[p] && Po
sQ[k - j]
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
```

```
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(d+ex^2\right)}{\sqrt{ax+bx^3+cx^5}}\,dx &= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{x}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \left(\frac{d\sqrt{x}}{\sqrt{a+bx^2+cx^4}}+\frac{ex^{5/2}}{\sqrt{a+bx^2+cx^4}}\right)dx}{\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(d\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}+\frac{\left(e\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{x^{5/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}\\
&= \frac{\left(d\sqrt{x}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{x}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{ax+bx^3+cx^5}}+\frac{\left(e\sqrt{x}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1\right.}{}\\
&= \frac{2dx^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4}}}}{}
\end{aligned}$$

Mathematica [A] time = 5.13468, size = 239, normalized size = 0.83

$$\frac{2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(7dx^2F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+3ex^4F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\right)}{21\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(7*d*x^2*AppellF1[3
/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] + 3*e*x^4*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*Sqrt[x*(a + b*x^
2 + c*x^4)])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x\left(ex^2+d\right)\frac{1}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}\,(ex^2+d)}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,(d+ex^2)}{\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x*(d + e*x**2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.123 $\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rubi [A] time = 0.0088066, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx &= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0182329, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3}\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] -(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x
^4])])/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.011, size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3x^2+3}\,\mathrm{Artanh}\left(\frac{(x^2-2)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3x^2+3}}\right)\frac{1}{\sqrt{x^6-3x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-3*x^4+3*x^2)^(1/2),x)
```

```
[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-
2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^6-3x^4+3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

Fricas [A] time = 1.26769, size = 142, normalized size = 3.16

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^3+2\sqrt{3}(x^3-2x)+2\sqrt{x^6-3x^4+3x^2}(\sqrt{3}+2)-6x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^6-3x^4+3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-3*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^6 - 3\,x^4 + 3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

3.124 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rubi [A] time = 0.0122878, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x\left(6-3x^2\right)}{\sqrt{3x^2-3x^4+x^6}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0033544, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3}\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x
^4])])/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.008, size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3x^2+3}\mathrm{Artanh}\left(\frac{\left(x^2-2\right)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3x^2+3}}\right)\frac{1}{\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4-3*x^2+3))^(1/2),x)
```

```
[Out] 1/6/(x^2*(x^4-3*x^2+3))^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^
2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\left(x^4-3x^2+3\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

Fricas [A] time = 1.3697, size = 142, normalized size = 3.16

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^3+2\sqrt{3}\left(x^3-2x\right)+2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}+2\right)-6x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(x^4 - 3\,x^2 + 3\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

3.125 $\int \frac{1}{\sqrt{1-\left(1-x^2\right)^3}} \, dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x\left(6-3x^2\right)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rubi [A] time = 0.0124409, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x\left(6-3x^2\right)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-\left(1-x^2\right)^3}} \, dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}} \, dx \\
&= -\text{Subst}\left(\int \frac{1}{12-x^2} \, dx, x, \frac{x\left(6-3x^2\right)}{\sqrt{3x^2-3x^4+x^6}}\right) \\
&= -\frac{\tanh^{-1}\left(\frac{x\left(6-3x^2\right)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0039223, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3}\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x
^4])])/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.003, size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3\,x^2+3}\mathrm{Artanh}\left(\frac{\left(x^2-2\right)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3\,x^2+3}}\right)\frac{1}{\sqrt{x^6-3\,x^4+3\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(-x^2+1)^3)^(1/2),x)
```

```
[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-
2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(x^2-1\right)^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((x^2 - 1)^3 + 1), x)
```

Fricas [A] time = 1.28434, size = 142, normalized size = 3.16

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^3+2\,\sqrt{3}\left(x^3-2\,x\right)+2\,\sqrt{x^6-3\,x^4+3\,x^2}\left(\sqrt{3}+2\right)-6\,x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1-\left(1-x^{2}\right)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - (1 - x**2)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(x^{2}-1\right)^{3}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((x^2 - 1)^3 + 1), x)
```

3.126 $\int \sqrt{3x^2-3x^4+x^6}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rubi [A] time = 0.0413086, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1903, 1107, 612, 619, 215}

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3x^2-3x^4+x^6}\,dx &= \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}}\\ &= \frac{\sqrt{3x^2-3x^4+x^6}\,\mathrm{Subst}\left(\int \sqrt{3-3x+x^2}\,dx,x,x^2\right)}{2x\sqrt{3-3x^2+x^4}}\\ &= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}}\,dx,x,x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\ &= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,-3+2x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\ &= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.034346, size = 70, normalized size = 0.81

$$\frac{x\left(4x^6-18x^4+30x^2+3\sqrt{x^4-3x^2+3}\sinh^{-1}\left(\frac{2x^2-3}{\sqrt{3}}\right)-18\right)}{16\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSinh[(-3 + 2
*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.007, size = 81, normalized size = 0.9

$$\frac{1}{16\,x}\sqrt{x^6-3\,x^4+3\,x^2}\left(4\,\sqrt{x^4-3\,x^2+3}x^2-6\,\sqrt{x^4-3\,x^2+3}+3\,\mathrm{Arcsinh}\left(1/3\,\sqrt{3}\left(2\,x^2-3\right)\right)\right)\frac{1}{\sqrt{x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-3*x^4+3*x^2)^(1/2),x)
```

```
[Out] 1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2-6*(x^4-3*x^2+3)^(1/
2)+3*arcsinh(1/3*3^(1/2)*(2*x^2-3)))/x/(x^4-3*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^6-3\,x^4+3\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

Fricas [A] time = 1.29569, size = 157, normalized size = 1.83

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\left(2\,x^2-3\right)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^6-3x^4+3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-3*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

Giac [A] time = 1.08852, size = 93, normalized size = 1.08

$$\frac{1}{16}\left(2\,\sqrt{x^4-3\,x^2+3}\left(2\,x^2-3\right)-3\,\log\left(-2\,x^2+2\,\sqrt{x^4-3\,x^2+3}+3\right)\right)\mathrm{sgn}\left(x\right)+\frac{3}{16}\left(2\,\sqrt{3}+\log\left(2\,\sqrt{3}+3\right)\right)\mathrm{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

3.127 $\int \sqrt{x^2 (3-3x^2+x^4)}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rubi [A] time = 0.0420436, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1996, 1903, 1107, 612, 619, 215}

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x^2\left(3-3x^2+x^4\right)}\,dx &= \int \sqrt{3x^2-3x^4+x^6}\,dx\\
&= \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}}\\
&= \frac{\sqrt{3x^2-3x^4+x^6}\,\text{Subst}\left(\int \sqrt{3-3x+x^2}\,dx,x,x^2\right)}{2x\sqrt{3-3x^2+x^4}}\\
&= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}}\,dx,x,x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\
&= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,-3+2x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\
&= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.0094265, size = 70, normalized size = 0.81

$$\frac{x\left(4x^6-18x^4+30x^2+3\sqrt{x^4-3x^2+3}\sinh^{-1}\left(\frac{2x^2-3}{\sqrt{3}}\right)-18\right)}{16\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSinh[(-3 + 2
*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.003, size = 81, normalized size = 0.9

$$\frac{1}{16\,x}\sqrt{x^2\left(x^4-3\,x^2+3\right)}\left(4\,\sqrt{x^4-3\,x^2+3}x^2-6\,\sqrt{x^4-3\,x^2+3}+3\,\text{Arcsinh}\left(1/3\,\sqrt{3}\left(2\,x^2-3\right)\right)\right)\frac{1}{\sqrt{x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^4-3*x^2+3))^(1/2),x)
```

```
[Out] 1/16*(x^2*(x^4-3*x^2+3))^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2-6*(x^4-3*x^2+3)^(
1/2)+3*arcsinh(1/3*3^(1/2)*(2*x^2-3)))/x/(x^4-3*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^4-3\,x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

Fricas [A] time = 1.28831, size = 157, normalized size = 1.83

$$-\frac{12\,x\log\left(-\frac{2\,x^{3}-3\,x-2\,\sqrt{x^{6}-3\,x^{4}+3\,x^{2}}}{x}\right)-8\,\sqrt{x^{6}-3\,x^{4}+3\,x^{2}}\left(2\,x^{2}-3\right)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09196, size = 93, normalized size = 1.08

$$\frac{1}{16}\left(2\,\sqrt{x^{4}-3\,x^{2}+3}\left(2\,x^{2}-3\right)-3\,\log\left(-2\,x^{2}+2\,\sqrt{x^{4}-3\,x^{2}+3}+3\right)\right)\mathrm{sgn}\left(x\right)+\frac{3}{16}\left(2\,\sqrt{3}+\log\left(2\,\sqrt{3}+3\right)\right)\mathrm{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

3.128 $\int \sqrt{1-(1-x^2)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rubi [A] time = 0.0411441, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1996, 1903, 1107, 612, 619, 215}

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rule 1903

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-\left(1-x^2\right)^3}\,dx &= \int \sqrt{3x^2-3x^4+x^6}\,dx\\
&= \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}}\\
&= \frac{\sqrt{3x^2-3x^4+x^6}\,\text{Subst}\left(\int \sqrt{3-3x+x^2}\,dx,x,x^2\right)}{2x\sqrt{3-3x^2+x^4}}\\
&= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}}\,dx,x,x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\
&= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}+\frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,-3+2x^2\right)}{16x\sqrt{3-3x^2+x^4}}\\
&= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.0033421, size = 70, normalized size = 0.81

$$\frac{x\left(4x^6-18x^4+30x^2+3\sqrt{x^4-3x^2+3}\sinh^{-1}\left(\frac{2x^2-3}{\sqrt{3}}\right)-18\right)}{16\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSinh[(-3 + 2
*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.003, size = 81, normalized size = 0.9

$$\frac{1}{16\,x}\sqrt{x^6-3\,x^4+3\,x^2}\left(4\,\sqrt{x^4-3\,x^2+3}x^2-6\,\sqrt{x^4-3\,x^2+3}+3\,\text{Arcsinh}\left(1/3\,\sqrt{3}\left(2\,x^2-3\right)\right)\right)\frac{1}{\sqrt{x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(-x^2+1)^3)^(1/2),x)
```

```
[Out] 1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2-6*(x^4-3*x^2+3)^(1/
2)+3*arcsinh(1/3*3^(1/2)*(2*x^2-3)))/x/(x^4-3*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(x^2-1\right)^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^2 - 1)^3 + 1), x)
```

Fricas [A] time = 1.29341, size = 157, normalized size = 1.83

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\left(2\,x^2-3\right)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{1-\left(1-x^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - (1 - x**2)**3), x)
```

Giac [A] time = 1.09643, size = 93, normalized size = 1.08

$$\frac{1}{16}\left(2\,\sqrt{x^4-3\,x^2+3}\left(2\,x^2-3\right)-3\,\log\left(-2\,x^2+2\,\sqrt{x^4-3\,x^2+3}+3\right)\right)\mathrm{sgn}\left(x\right)+\frac{3}{16}\left(2\,\sqrt{3}+\log\left(2\,\sqrt{3}+3\right)\right)\mathrm{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

3.129 $\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/Sqrt[a])
```

Rubi [A] time = 0.0158717, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/Sqrt[a])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0093191, size = 37, normalized size = 0.97

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]),x]

[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]/Sqrt[a])
```

Maple [A] time = 0.003, size = 35, normalized size = 0.9

$$-\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2),x)

[Out] -1/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36145, size = 273, normalized size = 7.18

$$\left[\frac{\log\left(-\frac{8\,abx+\left(b^2+4\,ac\right)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\left(acx^2+abx+a^2\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a
)*sqrt(a) + 8*a^2)/x^2)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)), x)
```

Giac [A] time = 1.0966, size = 47, normalized size = 1.24

$$\frac{2 \arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

3.130 $\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])
```

Rubi [A] time = 0.0216179, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx &= \int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx\\ &= -\left(2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0228697, size = 70, normalized size = 1.56

$$-\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

```
[Out] -((x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*
x^2])])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))]))
```

Maple [A] time = 0.008, size = 64, normalized size = 1.4

$$-x\sqrt{cx^2+bx+a}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{x^2\left(cx^2+bx+a\right)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2+b*x+a))^(1/2),x)
```

```
[Out] -1/(x^2*(c*x^2+b*x+a))^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2*a^
(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\left(cx^2+bx+a\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((c*x^2 + b*x + a)*x^2), x)
```

Fricas [A] time = 1.39367, size = 300, normalized size = 6.67

$$\left[\frac{\log\left(-\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}},\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\left(acx^3+abx^2+a^2x\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
```

```
b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10413, size = 80, normalized size = 1.78

$$-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+\frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

3.131 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x^3])]/Sqrt
[a])
```

Rubi [A] time = 0.0756258, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

```
[Out] -(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x^3])]/Sqrt
[a])
```

Rule 1997

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{ax+bx^2+cx^3}}\,dx$$
$$= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}(2a+bx)}{\sqrt{ax+bx^2+cx^3}}\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.033077, size = 72, normalized size = 1.53

$$-\frac{\sqrt{x}\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

```
[Out] -((Sqrt[x]*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[a]*Sqrt[x*(a + x*(b + c*x))]))
```

Maple [A] time = 0.011, size = 64, normalized size = 1.4

$$-\sqrt{x}\sqrt{cx^2+bx+a}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{x\left(cx^2+bx+a\right)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x)
```

```
[Out] -x^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2*
a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^2+bx+a)x}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)), x)
```

Fricas [A] time = 1.70371, size = 315, normalized size = 6.7

$$\left[\frac{\log\left(\frac{8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x-4\,\sqrt{cx^3+bx^2+ax}(bx+2\,a)\sqrt{a}\sqrt{x}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^3+bx^2+ax}(bx+2\,a)\sqrt{-a}\sqrt{x}}{2\left(acx^3+abx^2+a^2x\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^3 + b*x^2 +
a*x)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*
x^3 + b*x^2 + a*x)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^3 + a*b*x^2 + a^2*x)
)/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14507, size = 47, normalized size = 1.

$$\frac{2\,\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

3.132 $\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c*x^5])]/Sq
rt[a])
```

Rubi [A] time = 0.0899707, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

```
[Out] -(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c*x^5])]/Sq
rt[a])
```

Rule 1997

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^4+cx^5}}\,dx$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{3/2}(2a+bx)}{\sqrt{ax^3+bx^4+cx^5}}\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.0279648, size = 74, normalized size = 1.51

$$-\frac{x^{3/2}\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^3(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

```
[Out] -((x^(3/2)*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[a]*Sqrt[x^3*(a + x*(b + c*x))]))
```

Maple [A] time = 0.006, size = 66, normalized size = 1.4

$$-x^{\frac{3}{2}}\sqrt{cx^2+bx+a}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{x^3\left(cx^2+bx+a\right)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x)
```

```
[Out] -1/(x^3*(c*x^2+b*x+a))^(1/2)*x^(3/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{\left(cx^2+bx+a\right)x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3), x)
```

Fricas [A] time = 1.70724, size = 325, normalized size = 6.63

$$\left[\frac{\log\left(\frac{8\,abx^3+\left(b^2+4\,ac\right)x^4+8\,a^2x^2-4\,\sqrt{cx^5+bx^4+ax^3}(bx+2\,a)\sqrt{a}\sqrt{x}}{x^4}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^4+ax^3}(bx+2\,a)\sqrt{-a}\sqrt{x}}{2\left(acx^4+abx^3+a^2x^2\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*x^3 + (b^2 + 4*a*c)*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^5 + b*x^4
+ a*x^3)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^4)/sqrt(a), sqrt(-a)*arctan(1/2*sqr
t(c*x^5 + b*x^4 + a*x^3)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^4 + a*b*x^3 +
a^2*x^2))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**3*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15662, size = 47, normalized size = 0.96

$$\frac{2\,\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

3.133 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rubi [A] time = 0.0357734, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1114, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0122254, size = 44, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])
```

Maple [A] time = 0.004, size = 39, normalized size = 0.9

$$-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.46687, size = 294, normalized size = 6.68

$$\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{-a}}{2\left(acx^4+abx^2+a^2\right)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x), x)
```

3.134 $\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6])]/(2*Sqrt
[a])
```

Rubi [A] time = 0.0151489, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6])]/(2*Sqrt
[a])
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x^2\left(a+bx^2+cx^4\right)}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^4+cx^6}}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x\left(2a+bx^2\right)}{\sqrt{ax^2+bx^4+cx^6}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{x\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.0163743, size = 81, normalized size = 1.65

$$-\frac{x\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{x^2\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -(x*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2
 + c*x^4])])/(2*Sqrt[a]*Sqrt[x^2*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.005, size = 72, normalized size = 1.5

$$-\frac{x}{2}\sqrt{cx^4+bx^2+a}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x^2\left(cx^4+bx^2+a\right)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x)
```

```
[Out] -1/2/(x^2*(c*x^4+b*x^2+a))^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*a+b*
x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(cx^4+bx^2+a\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [A] time = 1.32455, size = 311, normalized size = 6.35

$$\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^6+bx^4+ax^2}(bx^2+2\,a)\sqrt{a}}{x^5}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^4+ax^2}(bx^2+2\,a)\sqrt{-a}}{2\,(acx^5+abx^3+a^2x)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^6 + b*x^4 +
 a*x^2)*(b*x^2 + 2*a)*sqrt(a))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c
*x^6 + b*x^4 + a*x^2)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^5 + a*b*x^3 + a^2*x))/a
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^4 + bx^2 + a)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2), x)
```

3.135 $\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*
Sqrt[a])
```

Rubi [A] time = 0.0672684, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

```
[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*
Sqrt[a])
```

Rule 1997

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.0195306, size = 83, normalized size = 1.63

$$-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{x(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

```
[Out] -(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
 b*x^2 + c*x^4])])/(2*Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.008, size = 72, normalized size = 1.4

$$-\frac{1}{2}\sqrt{x}\sqrt{cx^4+bx^2+a}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x(cx^4+bx^2+a)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x)
```

```
[Out] -1/2*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*
a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^4+bx^2+a)x}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)), x)
```

Fricas [A] time = 1.37217, size = 327, normalized size = 6.41

$$\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}(bx^2+2\,a)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\,\sqrt{a}},\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}(bx^2+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^5+abx^3+a^2x)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 +
 a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*
sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3
 + a^2*x))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^4 + bx^2 + a)x}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)), x)
```

3.136 $\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}} dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 + c*x^7])]/(
2*Sqrt[a])
```

Rubi [A] time = 0.0772904, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 + c*x^7])]/(
2*Sqrt[a])
```

Rule 1997

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !General
izedTrinomialMatchQ[u, x]
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx^2+cx^4\right)}}\,dx = \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^5+cx^7}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{3/2}\left(2a+bx^2\right)}{\sqrt{ax^3+bx^5+cx^7}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{x^{3/2}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.0194707, size = 85, normalized size = 1.6

$$-\frac{x^{3/2}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{x^3\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -(x^(3/2)*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
 b*x^2 + c*x^4])])/(2*Sqrt[a]*Sqrt[x^3*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.007, size = 74, normalized size = 1.4

$$-\frac{1}{2}x^{\frac{3}{2}}\sqrt{cx^4+bx^2+a}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x^3\left(cx^4+bx^2+a\right)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x)
```

```
[Out] -1/2/(x^3*(c*x^4+b*x^2+a))^(1/2)*x^(3/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((
2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{\left(cx^4+bx^2+a\right)x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [A] time = 1.36241, size = 338, normalized size = 6.38

$$\left[\frac{\log\left(-\frac{\left(b^2+4\,ac\right)x^6+8\,abx^4+8\,a^2x^2-4\,\sqrt{cx^7+bx^5+ax^3}\left(bx^2+2\,a\right)\sqrt{a}\sqrt{x}}{x^6}\right)}{4\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^7+bx^5+ax^3}\left(bx^2+2\,a\right)\sqrt{-a}\sqrt{x}}{2\left(acx^6+abx^4+a^2x^2\right)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^7 + b*x^5
 + a*x^3)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^6)/sqrt(a), 1/2*sqrt(-a)*arctan(
1/2*sqrt(c*x^7 + b*x^5 + a*x^3)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^6 + a
*b*x^4 + a^2*x^2))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**3*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{\left(cx^4 + bx^2 + a\right)x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3), x)
```

3.137 $\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/(2*Sqrt[3])
```

Rubi [A] time = 0.0268767, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
```

```
[Out] -ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/(2*Sqrt[3])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{3-3x+x^2}}\,dx, x, x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{3\left(2-x^2\right)}{\sqrt{3-3x^2+x^4}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{3-3x^2+x^4}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0091794, size = 40, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
```

```
[Out] -ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])]/(2*Sqrt[3])
```

Maple [A] time = 0.003, size = 31, normalized size = 0.8

$$-\frac{\sqrt{3}}{6}\text{Artanh}\left(\frac{\left(-3\,x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4-3\,x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4-3*x^2+3)^(1/2),x)
```

```
[Out] -1/6*3^(1/2)*arctanh(1/6*(-3*x^2+6)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [A] time = 1.6456, size = 27, normalized size = 0.68

$$-\frac{1}{6}\sqrt{3}\,\text{arsinh}\left(-\sqrt{3}+\frac{2\sqrt{3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)
```

Fricas [A] time = 1.31135, size = 131, normalized size = 3.28

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^2+2\sqrt{3}\left(x^2-2\right)+2\sqrt{x^4-3\,x^2+3}\left(\sqrt{3}+2\right)-6}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^2 + 2*sqrt(3)*(x^2 - 2) + 2*sqrt(x^4 - 3*x^2 + 3)*(sq
rt(3) + 2) - 6)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4-3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**4 - 3*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 - 3\,x^2 + 3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 - 3*x^2 + 3)*x), x)
```

3.138 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rubi [A] time = 0.0119649, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])
```

Rule 1996

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^2\left(3-3x^2+x^4\right)}}\,dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x\left(6-3x^2\right)}{\sqrt{3x^2-3x^4+x^6}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0082086, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3}\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x
^4])])/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0., size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3x^2+3}\operatorname{Artanh}\left(\frac{\left(x^2-2\right)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3x^2+3}}\right)\frac{1}{\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4-3*x^2+3))^(1/2),x)
```

```
[Out] 1/6/(x^2*(x^4-3*x^2+3))^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^
2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(x^4-3x^2+3\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

Fricas [A] time = 1.28909, size = 142, normalized size = 3.16

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^3+2\sqrt{3}\left(x^3-2x\right)+2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}+2\right)-6x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^4 - 3x^2 + 3)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

3.139 $\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/Sqrt[3])
```

Rubi [A] time = 0.0474514, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
```

```
[Out] -(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/Sqrt[3])
```

Rule 1997

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{x\left(3-3x+x^2\right)}}\,dx &= \int \frac{1}{\sqrt{x}\sqrt{3x-3x^2+x^3}}\,dx \\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{(6-3x)\sqrt{x}}{\sqrt{3x-3x^2+x^3}}\right)\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{3x-3x^2+x^3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0197291, size = 62, normalized size = 1.44

$$\frac{\sqrt{x}\sqrt{x^2-3x+3}\tanh^{-1}\left(\frac{\sqrt{3}(x-2)}{2\sqrt{x^2-3x+3}}\right)}{\sqrt{3}\sqrt{x\left(x^2-3x+3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
```

```
[Out] (Sqrt[x]*Sqrt[3 - 3*x + x^2]*ArcTanh[(Sqrt[3]*(-2 + x))/(2*Sqrt[3 - 3*x + x
^2])])/(Sqrt[3]*Sqrt[x*(3 - 3*x + x^2)])
```

Maple [A] time = 0.011, size = 50, normalized size = 1.2

$$\frac{\sqrt{3}}{3}\sqrt{x}\sqrt{x^2-3x+3}\,\mathrm{Artanh}\left(\frac{(-2+x)\sqrt{3}}{2}\frac{1}{\sqrt{x^2-3x+3}}\right)\frac{1}{\sqrt{x\left(x^2-3x+3\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x)
```

```
[Out] 1/3*x^(1/2)/(x*(x^2-3*x+3))^(1/2)*(x^2-3*x+3)^(1/2)*3^(1/2)*arctanh(1/2*(-2
+x)*3^(1/2)/(x^2-3*x+3)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(x^2-3x+3\right)x}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((x^2 - 3*x + 3)*x)*sqrt(x)), x)
```

Fricas [A] time = 1.39325, size = 132, normalized size = 3.07

$$\frac{1}{6}\sqrt{3}\log\left(\frac{7x^3+4\sqrt{3}\sqrt{x^3-3x^2+3x}(x-2)\sqrt{x}-24x^2+24x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((7*x^3 + 4*sqrt(3)*sqrt(x^3 - 3*x^2 + 3*x)*(x - 2)*sqrt(x)
- 24*x^2 + 24*x)/x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x*(x**2-3*x+3))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14413, size = 69, normalized size = 1.6

$$-\frac{1}{3}\sqrt{3}\log\left(\left|-x+\sqrt{3}+\sqrt{x^2-3x+3}\right|\right)+\frac{1}{3}\sqrt{3}\log\left(\left|-x-\sqrt{3}+\sqrt{x^2-3x+3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(abs(-x + sqrt(3) + sqrt(x^2 - 3*x + 3))) + 1/3*sqrt(3)*log
(abs(-x - sqrt(3) + sqrt(x^2 - 3*x + 3)))
```

3.140 $\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$

Optimal. Leaf size=70

$$-\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}(n-q)}$$

```
[Out] -(ArcTanh[(x^(q/2)*(2*a + b*x^(n - q)))/(2*Sqrt[a]*Sqrt[b*x^n + c*x^(2*n -
q) + a*x^q])]/(Sqrt[a]*(n - q)))
```

Rubi [A] time = 0.0666428, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
```

```
[Out] -(ArcTanh[(x^(q/2)*(2*a + b*x^(n - q)))/(2*Sqrt[a]*Sqrt[b*x^n + c*x^(2*n -
q) + a*x^q])]/(Sqrt[a]*(n - q)))
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{q/2}(2a+bx^{n-q})}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{n-q}\\ &= -\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{\sqrt{a}(n-q)}\end{aligned}$$

Mathematica [F] time = 0.392503, size = 0, normalized size = 0.

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
```

```
[Out] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int x^{-1+\frac{q}{2}}\frac{1}{\sqrt{bx^{n}+cx^{2n-q}+ax^{q}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
```

```
[Out] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q}+bx^{n}+ax^{q}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*q)/(b*x**n+c*x**(2*n-q)+a*x**q)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q}+bx^n+ax^q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```